Пихтовникова Татьяна БФЗ 244

№4 $p(x|\alpha) = \frac{\alpha}{x^{\alpha+1}}$, $x \geq 1$

$\hat{\alpha} = \arg\max \ln L(\hat{\alpha})$

$\mathcal{L}(\hat{\alpha}) = \ln L(\hat{\alpha})$

$L(\hat{\alpha}) = \prod_i^n \frac{\alpha}{x^{\alpha+1}} = \frac{n\alpha}{x^{\alpha+1}}$

$\mathcal{L}(\hat{\alpha}) = \ln n\alpha - \Sigma \ln x_i^{\alpha+1} =$

$= \ln n\alpha - (\alpha+1) \cdot \sum \ln x_i$.

$\mathcal{L}'(\hat{\alpha}) = \frac{n}{\alpha} - \sum \ln x_i = 0$

$$\hat{\alpha} = \frac{n}{\sum_{i=1}^{n} \ln x_i}$$

$\mathcal{L}''(\hat{\alpha}) = -\frac{n}{\alpha^2}\Big|_{\hat{\alpha}} = -\sum_{i=1}^{n} \ln x_i < 0$

$\Rightarrow \max$.

№2 $\vec{z} = (z_1, z_2)$: $\mu = 0$:

$\langle z_1^4 \rangle = 12$

$\langle z_2^6 \rangle = 15$

$\langle z_1^2 z_2^4 \rangle = 10$

т.к. это норм. распределение

$\langle z_1^4 \rangle = 3 \langle z_1^2 \rangle = 3\sigma_1^4$

$\langle z_2^6 \rangle = 3 \cdot 5\, \sigma_2^6 = 15\sigma_2^6$

$3\sigma_1^4 = 12 \Rightarrow \sigma_1^2 = 2$

$\sigma_2^2 = 1$

$\langle z_1^2 z_2^4 \rangle = 10$

1) 2 способа выбрать z_1 и 4 способа выбрать z_2

$\langle z_1^2 z_2^4 \rangle = \sigma_1^2 \cdot 3\sigma_2^4 + 6\sigma_2^2 \operatorname{cov}^2(z_1 z_2)$

$10 = 2 \cdot 3 + 6 \cdot \operatorname{cov}^2(z_1 z_2)$

$\operatorname{cov}(z_1 z_2) = \pm\sqrt{\frac{2}{3}}$

$$\hat{C} = \begin{pmatrix} 2 & \pm\sqrt{\frac{2}{3}} \\ \pm\sqrt{\frac{2}{3}} & 1 \end{pmatrix}$$

№1 усл. некоррелируемости

$\operatorname{cov}(z, v) = 0$

$\operatorname{cov}(z, v) = \operatorname{cov}(x\cos\varphi + y\sin\varphi,\ y\cos\varphi - x\sin\varphi) =$

$= \cos\varphi\cos\varphi \cdot \operatorname{cov}(x, y) - \sin\varphi \cos\varphi \cdot \sin\varphi\, DX + \sin\varphi \cdot \cos\varphi\, DY -$

$- \sin\varphi \sin\varphi \operatorname{cov}(x, y) = \cos^2\varphi - 4\cos\varphi \cdot \sin\varphi +$

$+ 2\sin\varphi\cos\varphi - \sin^2\varphi \;(=)$

из. матрицы ковариации

$$\hat{C} = \begin{array}{ccc} & DX & \operatorname{cov}(x,y) \\ & \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix} & \\ & \operatorname{cov}(x,y) & DY \end{array}$$

~~$= 2(\cos\varphi - \sin\varphi)^2 = 0$~~

$(=)\ \underbrace{(\cos^2\varphi - \sin^2\varphi)}_{\cos 2\varphi} - \underbrace{2\cos\varphi \cdot \sin\varphi}_{\sin 2\varphi} = 0$

$\operatorname{tg} 2\varphi = 1 \Rightarrow \varphi = \frac{\pi}{4} + k\pi$

$\mathbb{D}(Z) = 4\cos^2\varphi + 2\sin^2\varphi + 2\sin\varphi\cdot\cos\varphi\cdot 1 = 5$

$\mathbb{D}(V) = 4\sin^2\varphi + 2\cos^2\varphi - 2\sin\varphi\cos\varphi = 1$

При $\varphi = \frac{\pi}{4} + \pi k$

№3) $p(z) = Az, \quad 0 \leq z \leq 2$

$$\int_0^2 Az\,dz = 1 \Rightarrow A\cdot\frac{4}{2} = 1 \Rightarrow A = \frac{1}{2}$$

$$F(z) = \int_0^z \frac{1}{2}t\,dt = \frac{z^2}{4}$$

$$F_{max}(z) = \left(\frac{z^2}{4}\right)^3 = \frac{z^6}{64}$$

$$f_{max}(z) = \frac{6z^5}{64} = \frac{3z^5}{32}$$

$$\mathbb{E}[max] = \int_0^2 z\,\frac{3z^5}{32}\,dz = \frac{3}{32}\cdot\frac{2^7}{7} = \frac{12}{7}$$

Сорри, время вышло(((