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② $\langle z^{2n} \rangle = (2n-1)!!\, \sigma_z^{2n}$

$\langle x^6 \rangle = (6-1)!!\, \sigma_x^6 = \frac{5}{9}$ $\quad \sigma_x^6 = \frac{1}{9 \cdot 24} = \frac{1}{216}$ $\quad \sigma_x = \left(\frac{1}{216}\right)^{\frac{1}{6}}$

$\langle y^4 \rangle = (4-1)!!\, \sigma_y^4 = 3$ $\quad \sigma_y^4 = \frac{3}{6} = \frac{1}{2}$ $\quad \sigma_y = \left(\frac{1}{2}\right)^{\frac{1}{4}}$

Пусть a - число пар (x,x), b - число пар (y,y), c - число пар (x,y)

Тогда, для $\langle x^2 y^4 \rangle$:

$$\begin{cases} a+b+c = 3 \\ 2a+c = 2 \\ 2b+c = 4 \end{cases} \qquad \begin{matrix} (\mathrm{I})\ a=0 \quad b=1 \quad c=2 \\ (\mathrm{II})\ a=1 \quad b=2 \quad c=0 \end{matrix}$$

N - число разбиений:

$(\mathrm{I})\quad N = \frac{2!\,4!}{2^0 \cdot 1 \cdot 2 \cdot 1 \cdot 2} = \frac{48}{4} = 12$

$\mathrm{II}\quad N = \frac{2!\,4!}{2 \cdot 1 \cdot 2^2 \cdot 2 \cdot 1} = \frac{48}{16} = 3$

$\langle x^2 y^4 \rangle = 12\,(\sigma_x^2)^0 (\sigma_y^2)^1 (\sigma_{xy})^2 + 3\,(\sigma_x^2)^1 (\sigma_y^2)^2 (\sigma_{xy})^0 = 12\,\sigma_y^2 \sigma_{xy}^2 + 3\sigma_x^2 \sigma_y^4 \ominus$

коэфф. корреляции: $\rho = \frac{\mathrm{cov}(x,y)}{\sigma_x \sigma_y} \Rightarrow \mathrm{cov}(x,y) = \rho \cdot \left(\frac{1}{216}\right)^6 \left(\frac{1}{2}\right)^4$

$\mathrm{cov}(x,y) = \sigma_{xy} = \rho \left(\frac{1}{216}\right)^6 \left(\frac{1}{2}\right)^4$

$\ominus\ 12 \cdot \frac{1}{\sqrt{2}} \cdot \rho^2\, \frac{1}{\sqrt{2}} \cdot \left(\frac{1}{216}\right)^{\frac{1}{3}} + 3 \left(\frac{1}{216}\right)^{\frac{1}{3}} \frac{1}{2} = 3$

$2\rho^2 \cdot \left(\frac{1}{216}\right)^{\frac{1}{3}} = \left(1 - \left(\frac{1}{216}\right)^{\frac{1}{3}} \cdot \frac{1}{2}\right)$

$\rho^2 = \frac{1}{2}\left((216)^{\frac{1}{3}} - \frac{1}{2}\right)$

$\rho = \sqrt{\frac{1}{2}\left((216)^{\frac{1}{3}} - \frac{1}{2}\right)}$

④ $p(x|\alpha) = \alpha^{k} x^{k-1} \dfrac{e^{-\alpha x}}{\Gamma(k)}$

$$L(\alpha) = p(x_1, \dots, x_n | \alpha) = \prod_{i=1}^{n} \frac{\alpha^k}{\Gamma(k)} x^{k-1} e^{-\alpha x} = \left(\frac{\alpha^k}{\Gamma(k)}\right)^n \left(\sum_{i=1}^{n} x_i^{k-1}\right) \exp\left(-\alpha \sum_{i=1}^{n} x_i\right)$$

$$\ell(\alpha) = \ln L(\alpha) = nk \ln\alpha - n \ln\Gamma(k) + (k-1)\ln\sum_{i=1}^{n} x_i - \alpha \sum_{i=1}^{n} x_i$$

$$S_n = \sum_{i=1}^{n} x_i$$

$$\frac{d\ell}{d\alpha} = \frac{nk}{\alpha} - \cancel{2} S_n \Rightarrow \alpha = \frac{nk}{S_n} = \frac{nk}{\sum_{i=1}^{n} x_i}$$

Проверка:

$$\frac{d^2\ell}{d\alpha^2} = -\frac{nk}{\alpha^2} < 0 \quad \text{- максимум}$$

① $\hat{C} = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$

$$\left.\begin{array}{l} var(x) = 3 \\ var(y) = 2 \\ cov(x, y) = -1 \end{array}\right| \Rightarrow \begin{array}{l} x = x' - z \\ y^* = y' - z \end{array} \Rightarrow \begin{array}{l} var(x' - z) = 3 \\ var(y' - z) = 2 \\ cov(x' - z, y' - z) = -1 \Rightarrow \end{array}$$

$$\Rightarrow cov(x' - z, y' - z) = E[x'y' - z(x' + y') + z^2] - E[x' - z]E[y' - z] =$$

$$= E[x'y'] - E[zx'] - E[zy'] + E[z^2] - E[x' - z]E[y' - z]$$

③ $f(y) = \frac{1}{6} \quad 0 \le y \le 6 \quad F(y) = \frac{y}{6}$

$$f_n(y) = \frac{n!}{(k-1)!(n-k)!} [F(y)]^{k-1} [1 - F(y)]^{n-k} f(y)$$

$$\frac{4!}{(3-1)!(4-3)!} = 12$$

$$f_{Y_{(3)}}(y) = 12 \left(\frac{y}{6}\right)^2 \left(1 - \frac{y}{6}\right) \frac{1}{6} = \frac{y^2}{18}\left(1 - \frac{y}{6}\right) = \frac{y^2}{18} - \frac{y^3}{18 \cdot 6}$$

$$P(1 < [Y_{(3)} - 3]^2 < 9) \qquad Y_{(3)} \in [0, 6] \Rightarrow$$

$(Y_{(3)} - 3)^2 \le 9$ всегда выполняется

$$\Rightarrow P(1 < [Y_{(3)} - 3]^2 < 9)$$