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№4.

$p(x|\alpha) = \frac{\alpha}{x^{\alpha+1}}, \quad x \ge 1.$

$\cancel{L(\lambda) = \prod_{i=1}^{n} \alpha \cdot \frac{1}{x_i^{\alpha+1}} = \alpha^n \cdot}$

$\cancel{l(\lambda) = \ln L(\lambda) = n \ln\alpha \cdot}$

$L(\lambda) = \prod_{i=1}^{n} \alpha \cdot \frac{1}{x_i^{\alpha+1}} = \alpha^n \prod_{i=1}^{n} x_i^{-(\alpha+1)}$

$l(\alpha) = \ln L(\alpha) = n\ln\alpha - (\alpha+1)\sum_{i=1}^{n} \ln x_i$

Берем произв.

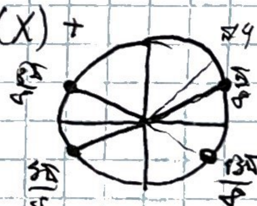
$l'(\alpha) = \frac{n}{\alpha} - \sum_{i=1}^{n} \ln x_i = 0 \Rightarrow \alpha^* = \frac{n}{\sum_{i=1}^{n} \ln x_i}$

$l''(\alpha) = -\frac{n}{\alpha^2} < 0$ — II произв. $\Rightarrow$ макс. Ответ: $\alpha^* = \frac{n}{\sum_{i=1}^{n} \ln x_i}$

№1.

$\cos\varphi = a, \quad \sin\varphi = b.$ $\quad C = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} var(X) & cov(XY) \\ cov(XY) & var(Y) \end{pmatrix}$

1) $Z = aX + bY, \quad V = -bX + aY$



$cov(Z,V) = 0 = cov(aX + bY, -bX + aY) = a(-b)\,var(X) +$
$+ a^2 cov(X,Y) - b^2 cov(Y,X) + ab\,var(Y)$

$var(X) = 4; \quad var(Y) = 2, \quad cov(X,Y) = 1.$

$cov(Z,V) = -ab \cdot 4 + a^2 - b^2 + ab \cdot 2 = (a^2 - b^2). = \cos^2\varphi - \sin^2\varphi =$
$= \cos 2\varphi - \sin 2\varphi.$

$\cos 2\varphi - \sin 2\varphi = 0 \Rightarrow \operatorname{tg} 2\varphi = 1 \Rightarrow 2\varphi = \frac{\pi}{4} + \pi n \Rightarrow \varphi = \frac{\pi}{8} + \frac{\pi n}{2}, \; n \in \mathbb{Z}$

Рассм. $[0, 2\pi) \Rightarrow \varphi = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}.$

2) $var(Z) = var(aX + bY) = 4a^2 + 2b^2 + 2ab.$ — Оценивание.

$var(V) = var(-bX + aY) = 4b^2 + 2a^2 - 2ab.$

Если $\cos 2\varphi = \sin 2\varphi = \frac{1}{\sqrt{2}}$, например, при $\varphi = \frac{\pi}{8}, \frac{9\pi}{8} \Rightarrow var(Z) = 3 + \sqrt{2}; \; var(V) = 3 - \sqrt{2}.$

Если $\cos 2\varphi = \sin 2\varphi = -\frac{1}{\sqrt{2}}$ при $\varphi = \frac{5\pi}{8}, \frac{13\pi}{8} \Rightarrow var(Z) = 3 - \sqrt{2};$
$var(V) = 3 + \sqrt{2}$

№2.

$\bar{z} = (z_1, z_2)$, $p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{x^2/2\sigma^2}$ $\mu = 0$.

$\langle z_1^4 \rangle = 12$, $\langle z_2^6 \rangle = 15$, $\langle z_1^2 z_2^4 \rangle = 10$.

для $\langle z^{2k} \rangle$: $E[z^{2k}] = (2k-1)!!\,\sigma^{2k}$.

$\langle z_1^4 \rangle$: $(2k-1)!! = 1 \cdot 3 = 3$; $3\sigma^4_{z_1^4} = 12 \Rightarrow \sigma^4_{z_1^4} = 4 \Rightarrow \sigma^2_{z_1^4} = 2$

$\langle z_2^6 \rangle$: $(2k-1)!! = 5 \cdot 3 \cdot 1 = 15$; $15\sigma^6_{z_2^6} = 15 \Rightarrow \sigma^2_{z_2^6} = 1$.

$E[z_1^2 z_2^4] = 3 \cdot 1 \cdot \sigma^2_{z_1} \cdot \sigma_{z_1 z_2} \cdot \sigma^2_{z_2} = 10$.

$10 = 3 \cdot 2 \cdot 1^2 + 12\sigma^2_{z_1 z_2} = 6 + 12\sigma^2_{z_1 z_2} \Rightarrow \sigma_{z_1 z_2} = \pm\frac{1}{\sqrt{3}}$.

$\hat{C}_1 = \begin{pmatrix} 2 & 1/\sqrt{3} \\ 1/\sqrt{3} & 1 \end{pmatrix}$ или $\hat{C}_2 = \begin{pmatrix} 2 & -1/\sqrt{3} \\ -1/\sqrt{3} & 1 \end{pmatrix}$.

№3.

$p(z) = Az$, $0 \le z \le 2$. z_1, z_2, z_3 - нез.

$1 = \int_0^2 Az\,dz = A \cdot \frac{z^2}{2}\Big|_0^2 = A \cdot \frac{4}{2} = 2A = 1 \Rightarrow A = \frac{1}{2} \Rightarrow p(z) = \frac{z}{2}$.

нормировка.

$F = P(z) = \int_0^z \frac{t}{2}\,dt = \frac{t^2}{4}\Big|_0^z = \frac{z^2}{4}$, $0 \le z \le 2$ - ф-ция распред. для одного z.

$M = \max(z_1, \dots, z_3)$

$F_M(m) = \mathbb{P}(M \le m) = \mathbb{P}(z_1 \le m, \dots, z_3 \le m) = F^3(m) = \left(\frac{m^2}{4}\right)^3 = \frac{m^6}{64}$

$p_M(m) = F_M'(m) = \frac{6m^5}{64} = \frac{3}{32} m^5$ $0 \le m \le 2$.

$E[M] = \int_0^2 m\,p_M(m)\,dm = \int_0^2 m \cdot \frac{3}{32} m^5\,dm = \frac{3}{32} \cdot \frac{1}{7} m^7\Big|_0^2 = \frac{3}{32} \cdot \frac{128}{7} = \frac{12}{7}$

$E[M^2] = \frac{3}{32}\left(\frac{m^8}{8}\right)\Big|_0^2 = \frac{3}{32} \cdot \frac{256}{8} = 3$.

$D[M] = E[M^2] - E^2[M] = 3 - \left(\frac{12}{7}\right)^2 = 3 - \frac{144}{49} = \frac{147-144}{49} = \frac{3}{49}$.