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№4.

$$p(x|\alpha) = \frac{\alpha^k x^{k-1} e^{-\alpha x}}{\Gamma(k)}$$

$$\cancel{L = \ln \prod_{i=1}^{n} \alpha^k x^{k-1}}$$

$$L = \ln \prod_{i=1}^{n} p(x_i|\alpha) = \ln \prod_{i=1}^{n} \frac{\alpha^k x_i^{k-1} e^{-\alpha x_i}}{\Gamma(k)} =$$

$$= \ln\left(\frac{\alpha^k}{\Gamma(k)} \prod_{i=1}^{n} x_i^{k-1} e^{-\alpha x_i}\right) = \ln\left(\frac{\alpha^k}{\Gamma(k)}\right) +$$

$$+ \ln\left(\prod_{i=1}^{n} x_i^{k-1} e^{-\alpha \sum_{i=1}^{n} x_i}\right) = \ln\left(\frac{\alpha^k}{\Gamma(k)}\right) + \ln\left(\prod_{i=1}^{n} x_i^{k-1}\right) - \alpha \sum_{i=1}^{n} x_i$$

$$\frac{dL}{d\alpha} = \frac{k\alpha^{k-1}}{\alpha^k} - \sum_{i=1}^{n} x_i$$

$$\frac{dL}{d\alpha} = 0 \ ; \quad \frac{k\alpha^{k-1}}{\alpha^k} - \sum_{i=1}^{n} x_i = 0$$

$$\frac{k}{\alpha} = \sum_{i=1}^{n} x_i$$

$$\frac{k}{\alpha} = X \rightarrow \alpha = \frac{k}{X}$$

$$\frac{d^2L}{d\alpha^2} = -\frac{k}{\alpha^2}$$

$$\left.\frac{d^2L}{d\alpha^2}\right|_{\alpha = \frac{k}{X}} = -\frac{k}{\frac{k^2}{X^2}} = -\frac{X^2}{k} = -\frac{\sum_{i=1}^{n} x_i}{k} < 0 \Rightarrow$$

$$\Rightarrow \alpha = \frac{k}{\sum_{i=1}^{n} x_i} \text{ — максимум}$$

Ответ: $\alpha = \dfrac{k}{\sum_{i=1}^{n} x_i}$

№1.

$$\hat{C} = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$$

Z - независимая от X, от Y

$C_{ij} = E[X_i X_j] - E[X_i]E[X_j]$

$E[X^2] - E[X]^2 = \sigma_x^2 = 3$

$E[Y^2] - E[Y]^2 = \sigma_Y^2 = 2$

$E[XY] - E[X]E[Y] = -1$

$X' = X + Z \qquad Y' = Y + Z$

$\sigma_{x'}^2 = E[X'^2] - E[X']^2 = ?$

$E[X'] = E[X] + E[Z]$

$X'^2 = X^2 + 2XZ + Z^2$

$E[X'^2] = E[X^2] + 2E[XZ] + E[Z^2]$

$E[X']^2 = (E[X] + E[Z])^2 = E[X]^2 + 2E[X]E[Z] + E[Z]^2$

$\sigma_{x'}^2 = \sigma_x^2 + 2E[XZ] - 2E[X]E[Z] + \sigma_z^2$

$\sigma_{x_1}^2 = \sigma_x^2 + \cancel{2E[X]E[Z]} - \cancel{2E[X]E[Z]} + \sigma_z^2 = \sigma_x^2 + \sigma_z^2$

$E[Y'^2] = E[Y^2] + 2E[YZ] + E[Z^2]$

$E[Y']^2 = E[Y]^2 + 2E[Y]E[Z] + E[Z]^2$

$\sigma_{Y'}^2 = \sigma_Y^2 + \sigma_z^2$

$\sigma_{x_1}^2 = 3 + \sigma_z^2$

$\sigma_{Y'}^2 = 2 + \sigma_z^2$

$\cancel{\sigma_z^2 = E[(X'-X)^2] E[Z^2] - E[Z]^2}$

$\cancel{E[Z^2] = E[X^2] + 2E[X]E[X'] + E[X'^2] = E[Y^2] - 2E[Y]E[Y'] + E[Y'^2]}$

$\cancel{E[Z]^2 = E[X]^2 - 2E[X][X'] + E[X']^2 = E[Y]^2 - 2E[Y]E[Y'] + E[Y']^2}$

$\cancel{\sigma_z^2 = \sigma_x^2 + \sigma_{x'}^2 - \sigma_Y^2 + \sigma_{Y'}^2}$

$\cancel{\sigma_z^2 = 3 + \sigma_{x'}^2 = 2 + \sigma_{Y'}^2}$

N2.

$E[X] = E[Y] = 0$

$E[X^6] = \frac{5}{9}$

$E[Y^4] = E[X^2Y^4] = 3$

$\tau(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y}; \; -1 \leq \tau \leq 1$

$cov(X,Y) = E((X-E(X))(Y-E(Y))) =$

$= E(XY) - E(X)E(Y)$

т.к. $E(X) = E(Y) = 0$, то

$cov(X,Y) = E[XY]$

$\cancel{E[X] = 0}$

$\cancel{E[X^6] = \frac{5}{9}}$

$\cancel{E[X^2Y^4] = \langle XXYYYY \rangle}$

$p(X) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\frac{X^2}{2\sigma_X^2}}$; $p(Y) = \frac{1}{\sqrt{2\pi\sigma_Y^2}} e^{-\frac{Y^2}{2\sigma_Y^2}}$

$E[X] = 0$

$E[X^6] = \frac{5}{9}$

$E[X^6] = 15\langle X^2\rangle\langle X^2\rangle\langle X^2\rangle = \frac{5}{9}$

$\langle X^2 \rangle = \frac{1}{27}$

$\cancel{E[X^6] = \sigma_X^6 = \frac{5}{9} \rightarrow \sigma_X^2 = \sqrt[3]{\frac{5}{9}}}$

$E[Y^4] = 3\langle Y^2 \rangle = 3$

$\cancel{E[Y^4] = \sigma_Y^4 = 3 \rightarrow \sigma_Y^2 = \sqrt{3}}$

$\langle Y^2 \rangle = 1$

$\cancel{\sigma_X^2 = E[X^2] - E[X]^2 = E[X^2] = \sqrt[3]{\frac{5}{9}}}$

$\sigma_X^2 = \frac{1}{27}$

$\cancel{\sigma_Y^2 = E[Y^2] - E[Y]^2 = E[Y^2] = \sqrt{3}}$

$\sigma_Y^2 = 1$

$\cancel{\langle X^2Y^4 \rangle = \langle XXYYYY \rangle = 3\langle X^2\rangle\langle Y^2\rangle\langle Y^2\rangle +}$

$\sigma_X = \frac{1}{3\sqrt{3}}$

$\cancel{+ 12\langle XY\rangle^2\langle Y^2\rangle = 3}$

$\sigma_Y = 1$

$\cancel{3\langle X^2\rangle\langle Y^2\rangle + 4\langle XY\rangle^2\langle Y^2\rangle = 1}$

$\cancel{\sqrt[3]{\frac{5}{9}} \cdot 3 + 4\langle XY\rangle^2\sqrt{3} = 1}$

$\cancel{\langle XY\rangle^2 = 1 - 3\sqrt[3]{\frac{5}{9}}}$