В. 58 Шкирик Павел 3 группа

№2 число слагаемых $\frac{n!}{(\frac{n}{2})!\, 2^{\frac{n}{2}}}$ $\langle z_2^2\rangle = \tau^2$ $\langle z_1^2\rangle = \sigma^2$

$$\langle z_1^4\rangle = \frac{4!}{2!\,4}\langle z_1^2\rangle\langle z_1^2\rangle \qquad (\langle z_1^2\rangle)^2 = (\sigma^2)^2 = 12/3$$

$\sigma^2 = 2$

$$\langle z_2^6\rangle = \frac{1\cdot2\cdot3\cdot4\cdot5\cdot6}{1\cdot2\cdot3\cdot8}\langle z_2^2\rangle^3 = 15\,(\tau^2)^3 = 15$$

$\tau^2 = 1$

$$\langle z_1^2 z_2^4\rangle = 3\langle z_1z_1\rangle\langle z_2z_2\rangle^2 + 4\cdot3\langle z_1z_2\rangle\langle z_1z_2\rangle\langle z_2z_2\rangle =$$

$$= 3\sigma^2(\tau^2)^2 + 12\,\mathrm{cov}^2\,\tau^2 = 10$$

$6 + 12\,\mathrm{cov}^2 = 10$

$12\,\mathrm{cov}^2 = 4$ $\mathrm{cov}^2 = \frac{1}{3}$ $\mathrm{cov} = \pm\frac{1}{\sqrt3}$

~~Ответ: $\pm\frac{1}{\sqrt3}$~~

Ответ: $\begin{pmatrix} 2 & \pm\frac{1}{\sqrt3} \\ \pm\frac{1}{\sqrt3} & 1\end{pmatrix}$

№3 $p(z) = Az$ $0 \leq z \leq 2$

$$\int_0^2 Az\,dz = 1$$

$$A\frac{z^2}{2}\Big|_0^2 = 2A \qquad A = \frac{1}{2}$$

$$F_z(z) = A\frac{z^2}{2} = \frac{z^2}{4}$$

кумул. ф-я

$$p_{a_{max}}(z) = n\,F_z^{n-1}(z)\,p_z(z) = n\left(\frac{z^2}{4}\right)^{n-1}\frac{z}{2} = n\left(\frac{z}{2}\right)^{2(n-1)+1} =$$

$n=3$

$$= n\left(\frac{z}{2}\right)^{2n-1} = 3\left(\frac{z}{2}\right)^5$$

$$\langle max\rangle = \int_0^2 z\cdot3\left(\frac{z}{2}\right)^5 dz = \frac{3}{2^5}\int_0^2 z^6\,dz = \frac{3}{2^5}\frac{z^7}{7}\Big|_0^2 = \frac{3\cdot2^2}{7} = \frac{12}{7}$$

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$$\langle max^2 \rangle = \int_0^2 z^2 \, 3\left(\frac{z}{2}\right)^5 dz = \frac{3}{2^5} \int_0^2 z^7 dz = \frac{3}{2^5} \left.\frac{z^8}{8}\right|_0^2 = \frac{3}{2^5} \cdot \frac{2^8}{2^3} = 3$$

$$D(max) = \langle max^2 \rangle - \langle max \rangle^2 = 3 - \frac{144}{49} = \frac{147-144}{49} = \frac{3}{49}$$

Ответ: $A = \frac{1}{2}$, $E = \frac{12}{7}$, $D = \frac{3}{49}$.

№4. $(x_1, x_2 \ldots x_n)$ $\quad p(x|\alpha) = \frac{\alpha}{x^{\alpha+1}}$ $\quad x \geq 1$

$$L(\alpha|(x_1 \ldots x_n)) = \prod_{1\leq i\leq n} p(x_i|\alpha)$$

$$L = \sum_{1\leq i\leq n} \ln(p(x_i|\alpha)) = \sum_{1\leq i\leq n} \ln\left(\frac{\alpha}{x_i^{\alpha+1}}\right) =$$

$$= n\ln\alpha - \sum_i \ln(x_i^{\alpha+1}) = n\ln\alpha - \sum_i (\alpha+1)\ln x_i$$

$$\frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} - \sum_i \ln x_i = 0$$

$$\frac{n}{\alpha} = \sum_i \ln x_i \qquad \alpha = \frac{n}{\sum_{1\leq i\leq n} \ln x_i}$$ — Ответ.

№5. $g(X,Y)$ $\quad \hat{C} = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}$

$$Z = X\cos\varphi + Y\sin\varphi$$

$$V = Y\cos\varphi - X\sin\varphi$$

$\langle X^2 \rangle = 4$ $\quad \langle Y^2 \rangle = 2$ $\quad \langle XY \rangle - \langle X \rangle \langle Y \rangle = 1$

$$\langle Z \rangle = \cos\varphi \langle X \rangle + \sin\varphi \langle Y \rangle$$

$$\langle V \rangle = \cos\varphi \langle Y \rangle - \sin\varphi \langle X \rangle$$

$$\langle Z^2 \rangle = \cos\varphi \langle X^2 \rangle + \sin\varphi \langle Y^2 \rangle$$

$$\langle V^2 \rangle = \cos\varphi \langle Y^2 \rangle - \sin\varphi \langle X^2 \rangle$$

$$Z \cdot V = (X\cos\varphi + Y\sin\varphi)(Y\cos\varphi - X\sin\varphi) =$$

$$= -\cos\varphi\sin\varphi X^2 + \sin\varphi\cos\varphi Y^2 + (\cos^2\varphi - \sin^2\varphi) XY$$

№1 (X,Y) $\hat{C} = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}$ Шкирин Павел

$$Z = X\cos\varphi + Y\sin\varphi$$
$$V = Y\cos\varphi - X\sin\varphi$$

$$\langle X^2\rangle - \langle X\rangle^2 = 4 \qquad \langle Y^2\rangle - \langle Y\rangle^2 = 2$$

$$\langle XY\rangle - \langle X\rangle\langle Y\rangle = 1$$

$$\langle Z\cdot V\rangle = \langle (X\cos\varphi + Y\sin\varphi)(Y\cos\varphi - X\sin\varphi)\rangle =$$

$$= -\sin\varphi\cos\varphi\langle X^2\rangle + \sin\varphi\cos\varphi\langle Y^2\rangle + (\cos^2\varphi - \sin^2\varphi)\langle XY\rangle$$

$$\langle Z\cdot V\rangle - \langle Z\rangle\cdot\langle V\rangle = 0$$

$$-\sin\varphi\cos\varphi\langle X^2\rangle + \sin\varphi\cos\varphi\langle Y^2\rangle + (\cos^2\varphi - \sin^2\varphi)\langle XY\rangle -$$
$$- (\cos\varphi\langle X\rangle + \sin\varphi\langle Y\rangle)(\cos\varphi\langle Y\rangle - \sin\varphi\langle X\rangle) =$$

$$= -\sin\varphi\cos\varphi(\overbrace{\langle X^2\rangle - \langle X\rangle^2}^{4}) + \sin\varphi\cos\varphi(\overbrace{\langle Y^2\rangle - \langle Y\rangle^2}^{2}) +$$
$$+ (\cos^2\varphi - \sin^2\varphi)(\underbrace{\langle XY\rangle - \langle X\rangle\langle Y\rangle}_{1}) =$$

$$= -2\sin 2\varphi + \sin 2\varphi + \cos 2\varphi = \cos 2\varphi - \sin 2\varphi = 0$$

$$2\varphi = \frac{\pi}{4} + \pi k$$

$$\boxed{\varphi = \frac{\pi}{8} + \frac{\pi k}{2} \quad k\in\mathbb{Z}}$$

$$\langle Z^2\rangle - \langle Z\rangle^2 = \langle(\cos\varphi X + Y\sin\varphi)^2\rangle - (\cos\varphi\langle X\rangle + \sin\varphi\langle Y\rangle)^2 =$$

$$= \cos^2\varphi\langle X^2\rangle + 2\sin\varphi\cos\varphi\langle XY\rangle + \sin^2\varphi\langle Y^2\rangle -$$
$$- \cos^2\varphi\langle X\rangle^2 - 2\sin\varphi\cos\varphi\langle X\rangle\langle Y\rangle - \sin^2\varphi\langle Y\rangle^2 =$$

$$= \cos^2\varphi(\mathbb{D}(X)) + 2\sin\varphi\cos\varphi(\mathrm{cov}(X,Y)) + \sin^2\varphi(\mathbb{D}(Y)) =$$

$$= 2\cdot 1 + 2\cos^2\varphi + 1\sin 2\varphi =$$

$= 2 + 1 + 2\cos^2\varphi - 1 + \sin^2 2\varphi = 3 + \cos 2\varphi + \sin 2\varphi = 3 \pm \cancel{\sqrt{2}} \sqrt{2}$

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~~cov (V, V~~ V

$\mathrm{cov}(Y\cos\varphi - X\sin\varphi, Y\cos\varphi - X\sin\varphi) =$

$= \cos^2\varphi \,\mathrm{cov}(Y,Y) - 2\sin\varphi\cos\varphi \,\mathrm{cov}(X,Y) + \sin^2\varphi \,\mathrm{cov}(X,X) =$

$= 2\cos^2\varphi - \sin 2\varphi + 4\sin^2\varphi = 2 - \sin 2\varphi + \overset{2\sin^2\varphi}{\cancel{2\sin^2\varphi}} =$

$= 2 - \sin 2\varphi - (1 - \cancel{2\sin^2\varphi}\, 2\sin^2\varphi) + 1 = 3 - \sin 2\varphi - \cos 2\varphi =$

$= 3 \mp \sqrt{2}$

Ответ: $\varphi = \frac{\pi}{8} + \frac{\pi k}{2} \quad k \in \mathbb{Z}$

$D(\overset{Z}{\cancel{X}}) = 3 \pm \sqrt{2}$

$D(\underset{V}{\cancel{W}}) = 3 \mp \sqrt{2}$