В.57 Чепурной Матвей
БФЗ-243

1. $\hat{C} = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$

$\Rightarrow Var(X) = 3$; $Var(Y) = 2$ $cov(X,Y) = -1$

$X' = X + Z$; $Y' = Y + Z$

$cov(X', Y') = 0$

$cov(X', Y') = cov(X + Z, Y + Z)$

$cov(X, Z) = cov(Y, Z) = 0$, т.к. независимы

$\Rightarrow cov(X', Y') = \underbrace{cov(X, Y)}_{-1} + \underbrace{cov(X, Z)}_{0} + \underbrace{cov(Y, Z)}_{0} + \underbrace{cov(Z, Z)}_{Var Z}$

$= -1 + Var Z \Rightarrow \underline{Var Z = 1}$

$Var X' = Var(X + Z) = \underbrace{Var X}_{3} + \underbrace{Var Z}_{1} + 2\underbrace{cov(X, Z)}_{0} = \underline{\underline{4}}$

$Var Y' = Var(Y + Z) = \underbrace{Var Y}_{2} + \underbrace{Var Z}_{1} + 2\underbrace{cov(Y, Z)}_{0} = \underline{\underline{3}}$

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2. $X, Y \sim N$, $E[X] = E[Y] = 0$

$\langle X^6 \rangle = \frac{5}{9}$; $\langle Y^4 \rangle = 3$; $\langle X^2 Y^4 \rangle = 3$

$\langle X^{2m} \rangle = \sigma_X^{2m} (2m-1)!! \Rightarrow \langle X^6 \rangle = 15 \sigma_x^6$;

$\langle Y^4 \rangle = 3\sigma_Y^4$, $\langle X^2 \rangle = \sigma_X^2$

$\Rightarrow 15\sigma_X^6 = \frac{5}{9} \Rightarrow \sigma_X^6 = \frac{1}{27} \Rightarrow \sigma_X = \pm\frac{\sqrt{3}}{3}$

$3\sigma_Y^4 = 3 \Rightarrow \sigma_Y = \pm 1$

Формула Вика

$\langle X^2 Y^4 \rangle = \langle X^2 \rangle \langle Y^4 \rangle + 2$

$3 \langle X^2 \rangle + 3\sigma_X^2 \sigma_Y^4 + 6\sigma_Y^2 \sigma_Y^2 \langle XY \rangle^2$

$\Rightarrow \langle XY \rangle = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow \rho = \frac{\langle XY \rangle}{\sigma_X \sigma_Y} = \frac{1/\sqrt{3}}{1 \cdot 1/\sqrt{3}} = \pm 1$

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3. Y_1, Y_2, Y_3, Y_4 независимы. $\sim R[0,6]$

$n=4 \Rightarrow f_{(3)}(y) = \frac{4!}{2!\,1!} F(y)^2 (1-F(y)) f$

$f(y) = \frac{1}{6} \Rightarrow F(y) = \frac{y}{6} \quad 0 \le y \le 6$

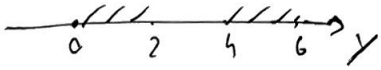
$f_{(3)}(y) = 12 \cdot \frac{y^2}{36} \left(1 - \frac{y}{6}\right) \frac{1}{6} = \frac{y^2(6-y)}{108}$

Чепурной Матвей

ГЗРЗ-243

$$P(1 < [Y_{(3)} - 3]^2 < 9) \overset{B.57}{}$$

$$\begin{bmatrix} |Y_{(3)} - 3| > 1 \\ |Y_{(3)} - 3| < 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 < Y_{(3)} < 6 \\ 0 < Y_{(3)} < 2 \end{bmatrix}$$



$$P = \int_0^2 f_{(3)}\, dy + \int_4^6 f_{(3)}\, dy = \frac{14}{27}$$

$$\int \frac{y^2(6-y)}{108} = \frac{1}{108}\left(2y^3 - \frac{y^4}{4}\right) + C$$

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4. $x_i := [\overline{1 \dots n}] \in \Gamma[\alpha, k]$

$$p(x|\alpha,k) = \frac{\alpha^k x^{k-1} e^{-\alpha x}}{\Gamma(k)} \qquad x \geq 0$$

Функц. правдоподобия

$$L(\alpha) = \prod_{i=1}^{n} \frac{\alpha^k x_i^{k-1} e^{-\alpha x_i}}{\Gamma(k)} \; ; \quad \ln L = \sum_{i=1}^{n} k \ln\alpha + (k-1)\ln x_i - \alpha x_i - \ln\Gamma(k)$$

$$\Rightarrow \ln L = nk\ln\alpha - \alpha \sum_{i=1}^{n} x_i + C$$

$$\frac{\partial \ln L}{\partial \alpha} = \frac{nk}{\alpha} - \sum x_i = 0 \Rightarrow \alpha = \frac{nk}{\sum x_i} = \frac{k}{\bar{x}}$$