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② X, Y по Гауссу, $\langle X^6\rangle = \frac{5}{9}$, $\langle Y^4\rangle = 3$, $\langle X^2Y^4\rangle = 3$ $\quad r = ?$

$\langle X\rangle = \langle Y\rangle = 0$.

$\langle x^{2n}\rangle = (2n-1)!!\,\sigma^{2n} \Rightarrow$ $\quad x: n=3 \Rightarrow (2n-1)!! = 5!! = 1\cdot3\cdot5 = 15 \Rightarrow 15\sigma_x^6 = \frac{5}{9} \Rightarrow$

$y: n=2 \Rightarrow 3!! = 3 \Rightarrow 3\sigma_y^4 = 3 \Rightarrow \sigma_y^2 = 1$ $\quad \Rightarrow \sigma_x^6 = \frac{1}{27} \Rightarrow \sigma^2 = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$

$\cancel{\langle X^2Y^4\rangle = 3 = 8\langle XY\rangle + 2\langle X^2\rangle + 6\langle Y^2\rangle = 8\langle XY\rangle + \frac{2}{3} + 6 = 3 \Rightarrow \langle XY\rangle = -\frac{11}{24}}$

$\cancel{r = \frac{cov(XY)}{\sigma_x^2\sigma_y^2}}$, $\cancel{cov(XY) = \langle XY\rangle - \langle X\rangle\langle Y\rangle = \langle XY\rangle}$

$\rho = \langle XY\rangle$

$\cancel{\Rightarrow r = -\frac{11}{24}\cdot\frac{3}{1} = -\frac{11}{8}}$

$\langle X^2Y^4\rangle = 3\langle X^2\rangle\langle Y^2\rangle^2 + 12\langle XY\rangle^2\langle Y\rangle^2 = 3\cdot\frac{1}{3}\cdot1 + 12\left(\rho^2\cdot\frac{1}{3}\right)$

$\Rightarrow 1 + 4\rho^2 = 3 \Rightarrow \rho = \pm\frac{1}{\sqrt{2}} = \langle XY\rangle$

$r = \frac{\langle XY\rangle}{\langle X\rangle^2\langle Y\rangle^2} = \pm\frac{1}{3\sqrt{2}}$

③ $p(x|a) = a^k x^{k-1}\frac{e^{-ax}}{\Gamma(k)}$ $\qquad L = (a, x_1, \dots, x_n) = \prod_{i=1}^{n}\frac{a^k x_i^{k-1}}{\Gamma(k)}e^{-ax_i} = \frac{a^{kn}}{\Gamma(k)^n}\left(\prod_{i=1}^{n}x_i^{k-1}\right)$

$\mathcal{L} = \ln(L(a)) = kn\ln a - a\sum_{i=1}^{n}x_i + const$

$\mathcal{L}' = \frac{kn}{a} - \sum_{i=1}^{n}x_i = 0 \Rightarrow a_* = \frac{kn}{\sum_{i=1}^{n}x_i} = \boxed{\frac{k}{\bar{x}}}$ $\qquad \mathcal{L}'' = -\frac{kn}{a^2} < 0$ ⊕ макс.

① $C_{ij} = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix} \Rightarrow \langle X^2\rangle = 3 = \sigma_x^2 \qquad \langle Y^2\rangle = 2 = \sigma_y^2 \qquad \langle XY\rangle = -1$

$X' = X + Z$
$Y' = Y + Z$

я думала, что получится просто выразить, но нет.

$\langle X', Y'\rangle = \langle (X+Z), (Y+Z)\rangle$, $\langle X, Z\rangle = \langle Y, Z\rangle = 0$ т.к. Z и X, Y независ

$\Rightarrow \langle X', Y'\rangle = \langle X, Y\rangle + \langle Z^2\rangle = 0 \Rightarrow \langle Z^2\rangle = 1.$

$\langle X'^2\rangle = \langle (X+Z)^2\rangle = \langle X^2\rangle + 2\langle X, Z\rangle + \langle Z^2\rangle = 3 + 1 = 4.$

$\langle Y'\rangle = \langle (Y+Z)^2\rangle = \langle Y^2\rangle + \langle Z^2\rangle = 2 + 1 = 3$