

② X, Y — гауссы, $\langle X^6 \rangle = \frac{5}{3}$, $\langle Y^4 \rangle = 3$, $\langle X^2 Y^4 \rangle = 3$ $\rho = ?$

$$\langle X \rangle = \langle Y \rangle = 0$$

$$X: n=3 \Rightarrow \langle X^{2n} \rangle = (2n-1)!! \sigma^2 = 5!! = 1 \cdot 3 \cdot 5 = 15 \Rightarrow \sigma_X^2 = \frac{5}{3}$$

$$Y: n=2 \Rightarrow \langle Y^4 \rangle = 3!! = 3 \Rightarrow \sigma_Y^2 = 1 \Rightarrow \sigma_X^2 = \frac{5}{3} \Rightarrow \sigma_X = \sqrt{\frac{5}{3}}$$

$$\langle X^2 Y^4 \rangle = 3 = 8 \langle X^2 Y^2 \rangle + 2 \langle X^4 \rangle + 6 \langle Y^4 \rangle = 8 \langle X^2 Y^2 \rangle + 2 \cdot \frac{5^2}{3} + 6 \cdot 3 = 3 \Rightarrow \langle X^2 Y^2 \rangle = \frac{1}{24}$$

$$\rho = \frac{\langle XY \rangle}{\sigma_X \sigma_Y} \Rightarrow \langle XY \rangle = \rho \sigma_X \sigma_Y = \rho \sqrt{\frac{5}{3}} \cdot 1 = \rho \sqrt{\frac{5}{3}}$$

$$\Rightarrow \rho = \frac{\langle XY \rangle}{\sqrt{\frac{5}{3}}} = \frac{1}{\sqrt{24}} = \frac{1}{2\sqrt{6}}$$

$$\rho = \langle XY \rangle$$

$$\langle X^2 Y^4 \rangle = 3 \langle X^2 \rangle \langle Y^4 \rangle + 12 \langle X^2 Y^2 \rangle \langle Y^2 \rangle = 3 \cdot \frac{5}{3} \cdot 3 + 12 \langle X^2 Y^2 \rangle \cdot 1 = 15 + 12 \langle X^2 Y^2 \rangle = 3 \Rightarrow \langle X^2 Y^2 \rangle = \frac{1}{24}$$

$$\Rightarrow 1 + 4\rho^2 = 3 \Rightarrow \rho = \pm \frac{1}{\sqrt{2}} \neq \langle XY \rangle$$

$$\rho = \frac{\langle XY \rangle}{\sigma_X \sigma_Y} = \pm \frac{1}{\sqrt{2}} \neq \frac{1}{2\sqrt{6}}$$

③ $p(x|a) = a^k x^{k-1} \frac{e^{-ax}}{\Gamma(k)}$ $L = (a, x_1, \dots, x_n) = \prod_{i=1}^n \frac{a^k x_i^{k-1}}{\Gamma(k)} e^{-ax_i} = \frac{a^{kn}}{\Gamma(k)^n} \left(\prod_{i=1}^n x_i^{k-1} \right) e^{-a \sum_{i=1}^n x_i}$

$$L = \ln(L(a)) = kn \ln a - a \sum_{i=1}^n x_i + \text{const}$$

$$L' = \frac{kn}{a} - \sum_{i=1}^n x_i = 0 \Rightarrow a_x = \frac{kn}{\sum_{i=1}^n x_i} = \frac{k}{\bar{x}} \quad L'' = -\frac{kn}{a^2} < 0 \quad \text{max}$$

$$\textcircled{1} C_{12} = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix} \Rightarrow \begin{matrix} \langle X^2 \rangle = 3 \\ \sigma_x^2 \end{matrix} \quad \begin{matrix} \langle Y^2 \rangle = 2 \\ \sigma_y^2 \end{matrix} \quad \langle XY \rangle = -1$$

$$\begin{cases} X' = X + Z \\ Y = Y + Z \end{cases}$$

~~$$\begin{aligned} \langle (X' - Z)^2 \rangle &= -1 \\ \Rightarrow \langle X'^2 - X'Z + Z^2 \rangle &= -1 = \langle X'^2 \rangle - \langle X'Z \rangle + \langle Z^2 \rangle \\ \langle (X' - Z)^2 \rangle &= 3 = \langle X'^2 \rangle - 2\langle X'Z \rangle + \langle Z^2 \rangle \\ \langle (Y' - Z)^2 \rangle &= 2 = \langle Y'^2 \rangle - 2\langle Y'Z \rangle + \langle Z^2 \rangle \end{aligned}$$~~

я думаю, что
получится
просто выразить,
но нет

~~$$\begin{aligned} \langle Z^2 \rangle &= -1 - \langle X'Y' \rangle + \langle X'Z \rangle + \langle Y'Z \rangle \\ \Rightarrow 3 &= \langle X'^2 \rangle - 2\langle X'Z \rangle - 1 - \langle X'Y' \rangle + \langle X'Z \rangle + \langle Y'Z \rangle \end{aligned}$$~~

~~$$\begin{aligned} 1) \langle X'Z \rangle &= -1 - \langle X'Y' \rangle + \langle Y'Z \rangle + \langle Z^2 \rangle \\ \Rightarrow 1 &= \langle X'^2 \rangle + 2 + 2\langle X'Y' \rangle - 2\langle Y'Z \rangle + 2\langle Z^2 \rangle \end{aligned}$$~~

~~$$2) -1 = \langle X'Y \rangle + \langle X'Z \rangle + \langle Y'Z \rangle + \langle Z^2 \rangle - \langle X'Z \rangle - \langle Z^2 \rangle - \langle Y'Z \rangle - \langle Z^2 \rangle$$~~

~~$$\begin{aligned} \langle X'Y' \rangle &= 0 = \langle (X+Z)(Y+Z) \rangle = \langle XY + XZ + YZ + Z^2 \rangle = \langle XY \rangle + \langle XZ \rangle + \\ &+ \langle YZ \rangle + \langle Z^2 \rangle = 0 \end{aligned}$$~~

~~$$\begin{aligned} 1) \langle X'Z \rangle &= 1 - \langle Y'Z \rangle + \langle Z^2 \rangle \Rightarrow 3 = \langle X'^2 \rangle - 2 + 2\langle Y'Z \rangle - 2\langle Z^2 \rangle + \langle Z^2 \rangle \Rightarrow \\ \Rightarrow 5 &= \langle X'^2 \rangle + 2\langle Y'Z \rangle - \langle Z^2 \rangle \Rightarrow \langle Y'Z \rangle = \frac{5 - \langle X'^2 \rangle + \langle Z^2 \rangle}{2} \\ \Rightarrow 2 &= \langle Y'^2 \rangle - 5 + \langle X'^2 \rangle - \langle Z^2 \rangle \end{aligned}$$~~

~~$$\begin{aligned} \text{cov}(X', Y') &= \text{cov}(X+Z, Y+Z) = \text{cov}(X, Y) + \text{cov}(X, Z) + \text{cov}(Y, Z) + \text{cov}(Z, Z) \\ \text{cov}(X, Z) &= \text{cov}(Y, Z) = 0 \Rightarrow \text{cov}(X', Y') = \text{cov}(X, Y) = 0 \end{aligned}$$~~

$$\langle X', Y' \rangle = \langle (X+Z), (Y+Z) \rangle = 0, \quad \langle X, Z \rangle = \langle Y, Z \rangle = 0 \quad \text{так } Z \text{ и } X, Y \text{ независимы}$$

$$\Rightarrow \langle X', Y' \rangle = \langle X, Y \rangle + \langle Z^2 \rangle = 0 \Rightarrow \langle Z^2 \rangle = 1$$

$$\langle X'^2 \rangle = \langle (X+Z)^2 \rangle = \langle X^2 \rangle + 2\langle XZ \rangle + \langle Z^2 \rangle = 3 + 1 = 4$$

$$\langle Y' \rangle = \langle (Y+Z)^2 \rangle = \langle Y^2 \rangle + \langle Z^2 \rangle = 2 + 1 = 3$$