Иванов А. А. – БФЗ-241

Вар 52

4. $(x_1, \ldots, x_n)$

$$p(x|\alpha) = \frac{\alpha}{x^{\alpha+1}}, \quad x \geq 1$$

$$L = \ln \prod_i p(x_i|\alpha) = \ln \prod_i \left(\frac{\alpha}{x_i^{\alpha+1}} \right) =$$

$$= \sum_i \left(\ln \alpha - \ln(x_i) \cdot (\alpha+1) \right) =$$

$$= n \ln \alpha - \left(\sum_i^n \ln(x_i) \right) (\alpha+1)$$

$$\hat{\alpha}_{ML} = \arg\max_\theta L(\theta | \overline{x})$$

$$L' = \frac{n}{\alpha^*} - \sum_i^n \ln x_i = 0$$

$$\alpha^* = \frac{n}{\sum_i^n \ln x_i}$$

$$L'' = -\frac{n}{\alpha^{+2}} < 0 \quad \forall \alpha$$

$$\boxed{\hat{\alpha}_{ML} = \frac{n}{\sum_i^n \ln x_i}}$$

3. Z_1, Z_2, Z_3

$$p(z) = Az, \quad 0 \le z \le 2$$

$$A\int_0^2 z\,dz = 1 \Rightarrow A = \frac{1}{2}$$

$$p(z) = \frac{z}{2}$$

$$\cancel{\max\{z_1, z_2, z_3\} =}$$

$$\cancel{p_{max}(z) = 3\binom{2}{2} F(z)\cdot p(z)}$$

$$P(\max\{z_1, z_2, z_3\} \le z) = P(z_1 \le z)\,P(z_2 \le z)\,P(z_3 \le z) =$$

$$= F^3(z)$$

$$p_{max}(z) = 3F^2(z)\cdot p(z)$$

$$F(z) = \int_{-\infty}^{z} p(z')\,dz' = \begin{cases} 0, & z < 0 \\ \frac{z^2}{4}, & z \in [0; 2] \\ 1, & z > 2 \end{cases}$$

$$p_{max}(z) = \begin{cases} \left(\frac{z^2}{4}\right)^2 \cdot 3 \cdot \frac{z}{2}; & 0 \le z \le 2 \\ 0, & \text{иначе} \end{cases}$$

$$\langle \max\{z_1, z_2, z_3\}\rangle = \int_0^2 z \cdot \frac{3}{32} z^5 \, dz =$$

$$= \frac{z^7}{7} \cdot \frac{3}{32}\Big|_0^2 = \frac{2^7 \cdot 3}{7 \cdot 2^5} = \frac{12}{7}$$

$$\langle \max^2\{z_1, z_2, z_3\}\rangle = \int_0^2 z^2 \frac{3}{32} z^5 \, dz =$$

$$= \frac{z^8}{8}\Big|_0^2 \cdot \frac{3}{32} = \frac{2^3 \cdot 3}{8} = 3$$

$$\sigma^2_{max} = \langle \max^2\rangle - \langle \max\rangle^2 = 3 - \frac{144}{49} =$$

$$= \frac{3}{49}$$

$$\boxed{\begin{aligned} A &= \frac{1}{2} \\ \sigma^2_{max} &= \frac{3}{49} \\ \mu_{max} &= \frac{12}{7} \end{aligned}}$$

№2. $\vec{z} = (z_1, z_2)$

$\langle z_1^4\rangle = 12$; $\langle z_2^6\rangle = 15$; $\langle z_1^2 z_2^4\rangle = 10$

cov − ? $\langle \vec{x}\rangle = 0$

$$\langle z_1^4 \rangle = \langle z_1 \cdot z_1 \cdot z_1 \cdot z_1 \rangle =$$

$$= \langle z_1 z_1 \rangle \langle z_1 z_1 \rangle + \langle z_1 z_1 \rangle \langle z_1 \cdot z_1 \rangle + \langle z_1 \cdot z_1 \rangle \langle z_1 z_1 \rangle$$

$$= 3 \langle z_1^2 \rangle^2 = 12$$

$$\langle z_1^2 \rangle = \pm 2$$

$$\langle z_2^6 \rangle = \langle z_2 z_2 \rangle \langle z_2 z_2 \rangle \langle z_2 z_2 \rangle + \ldots =$$

$$= 5 \langle z_2^2 \rangle \left(\langle z_2 z_2 \rangle \langle z_2 z_2 \rangle \cdot 3 \right) =$$

$$= 15 \langle z_2^2 \rangle^3 = 15$$

$$\langle z_2^2 \rangle^3 = 1 \Rightarrow \langle z_2^2 \rangle = 1$$

$$\langle z_1^2 z_2^4 \rangle = \langle z_1 z_1 \rangle \cdot 3 \langle z_2^2 \rangle^2 +$$

$$+ 4 \langle z_1 z_2 \rangle \left(\langle z_1 z_2 \rangle \langle z_2 z_2 \rangle \cdot 3 \right) =$$

$$= 3 \underbrace{\langle z_1^2 \rangle}_{\pm 2} \underbrace{\langle z_2^2 \rangle^2}_{1} + 12 \langle z_1 z_2 \rangle^2 \underbrace{\langle z_2^2 \rangle}_{1} = 10$$

$$\pm 6 + 12 \langle z_1 z_2 \rangle^2 = 10$$

$$\left[\begin{array}{l} 12 \langle z_1 z_2 \rangle^2 = 4 \\ 12 \langle z_1 z_2 \rangle^2 = 16 \end{array} \right.$$

$$\left[\begin{array}{l} \langle z_1 z_2 \rangle = \pm \sqrt{\frac{1}{3}} \\ \langle z_1 z_2 \rangle = \pm \sqrt{\frac{4}{3}} \end{array} \right.$$

$$\hat{C}(\bar{z}) = \begin{pmatrix} E[z_1^2]-0 & E[z_1 z_2]-0 \\ E[z_2 z_1]-0 & E[z_2^2]-0 \end{pmatrix}$$

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1)

$E[z_1^2] = 2$

$E[z_2^2] = 1$

$E[z_2 z_1] = \pm\sqrt{\frac{7}{3}}$

2)

$E[z_2^2] = 1$

$E[z_1^2] = -2$

$E[z_2 z_1] = \pm\sqrt{\frac{4}{3}}$

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1. $\hat{C} = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix} \Rightarrow \begin{cases} E[X^2] - E[X]^2 = 4 \\ E[XY] - E[X]E[Y] = 1 \\ E[Y^2] - E[Y]^2 = 2 \end{cases}$

$Z = X\cos\varphi + Y\sin\varphi$

$V = Y\cos\varphi - X\sin\varphi$ — некоррелированы

$\mathrm{cov}(Z,V) = E[ZV] - E[Z]E[V] = 0$

$E[ZV] = E[Z]\cdot E[V]$

$E[(x\cos\varphi + y\sin\varphi)(y\cos\varphi - x\sin\varphi)] =$

$= E[xy\cos^2\varphi - x^2\cos\varphi\sin\varphi +$

$+ y^2\sin\varphi\cos\varphi - yx\sin^2\varphi]$

$E[Z] = E[X\cos\varphi + Y\sin\varphi] = \cos\varphi E[X] + \sin\varphi E[Y]$

$E[V] = E[Y]\cos\varphi - E[X]\sin\varphi$

$$E[Z]E[V] = \cos^2\varphi E[X]E[Y] + \sin\varphi\cos\varphi \cancel{EX} E[Y]^2 +$$

$$+ E[X]^2 \cos\varphi\sin\varphi(-1) + E[X]E[Y]\sin^2\varphi(-1) =$$

$$= E[X]E[Y](\cos^2\varphi - \sin^2\varphi) + \cancel{E[Y]}$$

$$+ \sin\varphi\cos\varphi\left(E[Y]^2 - E[X]^2\right)$$

$$E[ZV] = E[xy](\cos^2\varphi - \sin^2\varphi) +$$

$$+ \cos\varphi\sin\varphi\left(\cancel{E[X]}\, E[Y^2] - E[X^2]\right)$$

Используя ~~EX EY~~ $E[ZV] = E[Z]E[V]$

$$\underbrace{\left(E[XY] - E[X]E[Y]\right)}_{1}(\cos^2\varphi - \sin^2\varphi) +$$

$$+ \cos\varphi\sin\varphi\left(\underbrace{E[Y^2] - E[Y]^2}_{2} - \underbrace{\left(E[X^2] - E[X]^2\right)}_{4}\right) = 0$$

$$\boxed{\cos^2\varphi - \sin^2\varphi - 2\cos\varphi\sin\varphi = 0}$$

$$\cos(2\varphi) - \sin(2\varphi) = 0$$

$$\cos\Theta = \sin\Theta$$

$$tg\,\Theta = 1$$

$$\cancel{\left[\begin{array}{l}\Theta = \frac{\pi}{4} + 2\pi \\ \Theta = \frac{5\pi}{4}\end{array}\right.}$$

$$\Theta = \frac{\pi}{4} + \pi n, \ n \in \mathbb{Z}$$

$$\boxed{\varphi = \frac{\pi}{8} + \frac{\pi n}{2}, \ n \in \mathbb{Z}}$$

$$\sigma_z^2 = \langle z^2\rangle - \langle z\rangle^2 =$$

$$= \langle (x\cos\varphi + y\sin\varphi)^2\rangle - \langle x\cos\varphi + y\sin\varphi\rangle^2 =$$

$$= \langle x^2\rangle\cos^2\varphi + \langle y^2\rangle\sin^2\varphi + 2\langle yx\rangle\cos\varphi\sin\varphi -$$

$$- \langle x\rangle^2\cos^2\varphi - \langle y\rangle^2\sin^2\varphi - 2\cos\varphi\sin\varphi\langle y\rangle\langle x\rangle =$$

$$= 4\cos^2\varphi + 2\sin^2\varphi + 2\cos\varphi\sin\varphi =$$

$$= 2\left(2\cos^2\varphi + \sin^2\varphi + \cos\varphi\sin\varphi\right)$$

$$\sigma_v^2 = \langle v^2\rangle - \langle v\rangle^2 =$$

$$= \langle y^2\cos^2\varphi + x^2\sin^2\varphi - 2yx\cos\varphi\sin\varphi\rangle -$$

$$- \langle y\rangle^2\cos^2\varphi - \langle x\rangle^2\sin^2\varphi + 2\cos\varphi\sin\varphi\langle x\rangle\langle y\rangle =$$

$$= \cos^2\varphi\cdot 2 + \sin^2\varphi\cdot 4 - 2\cos\varphi\sin\varphi =$$

$$= 2\left(\cos^2\varphi + 2\sin^2\varphi - \cos\varphi\sin\varphi\right)$$

$$\boxed{\begin{aligned} \sigma_z^2 &= 2\left(2\cos^2\varphi + \sin^2\varphi + \cos\varphi\sin\varphi\right) \\ \sigma_v^2 &= 2\left(\cos^2\varphi + 2\sin^2\varphi - \cos\varphi\sin\varphi\right) \\ \varphi &= \frac{\pi}{8} + \frac{\pi n}{2}, \; n \in \mathbb{Z} \end{aligned}}$$