№4

Костельный

1гр.

$$P(x|\lambda) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{\Gamma(k)}$$

$$L = \sum_{i=1}^{n} \ln \frac{\lambda_i^k x_i^{k-1} e^{-\lambda x_i}}{\Gamma_i(k)} = \sum_{i=1}^{n} \ln \lambda_i^k + \ln \frac{x_i^{k-1}}{\Gamma_i(k)} - \lambda x_i =$$

$$= nk \ln \lambda + \sum_{i=1}^{n} \ln \frac{x_i^{k-1}}{\Gamma_i(k)} - \lambda \sum_{i=1}^{n} x_i$$

$$L' = nk \frac{1}{\lambda} - \sum_{i=1}^{n} x_i \Rightarrow \boxed{\lambda^* = \frac{nk}{\sum_{i=1}^{n} x_i}}$$

$$L'' = -\frac{nk}{\lambda^2} < 0 \Rightarrow \lambda^* - \max$$

№3

$y_1, y_2, y_3, y_4 \sim$ $p_y = \frac{1}{6}$

$$F(y) = \int_0^y \frac{1}{6}\, dy = \frac{y}{6}$$

$$P_{(3)}(y) = 4 \cdot \frac{3!}{2!(3-2)!} \left(\frac{y}{6}\right)^2 \left(1 - \frac{y}{6}\right) \frac{1}{6} =$$

$$= 12 \cdot \frac{1}{6} \cdot \frac{y^2}{36}\left(1 - \frac{y}{6}\right) = \frac{y^2}{18}\left(1 - \frac{y}{6}\right) = \boxed{\frac{y^2}{18} - \frac{y^3}{108}}$$