Андронов Марк. БФЗ-241

Вариант - 50

№2.

$\vec{z} = (z_1, z_2) \sim N(0, \sigma^2)$

$\langle z_1^4 \rangle = \mathbb{E}[z_1^4] = 12 = 3\sigma_{z_1}^4$, т.к. $\mathbb{E}[X^{2k}] = (2k-1)!!\,\sigma^{2k}$

$\sigma_{z_1}^4 = 4 \Rightarrow \sigma_{z_1}^2 = 2$

$\langle z_2^6 \rangle = \mathbb{E}[z_2^6] = 15 = 15\sigma_{z_2}^6 \Rightarrow \sigma_{z_2}^6 = 1 \Rightarrow \sigma_{z_2}^2 = 1$

$Cov(z_1, z_2) = \mathbb{E}[\overset{z_1 z_2}{\cancel{xy}}] - \underbrace{\mathbb{E}[\overset{z_1}{\cancel{x}}]}_{\to 0}\,\underbrace{\mathbb{E}[\overset{z_2}{\cancel{y}}]}_{\to 0}$

$\mathbb{E}[z_1^2 z_2^4] = 10$

По теореме Вика:

Пусть k - количество $z_1 z_2$ комбинаций

$a = 2$, $b = 4$, $k = \min(a, b)$; $\min(a, b) - 2$; ...

пока k - чётное всё работает

$k = 0$ (0 z_1, z_2)

$\mathbb{E}[z_1^2 z_2^4] = \mathbb{E}[z_1^2] + \mathbb{E}[z_2^2] + \mathbb{E}[z_2^2] = \sigma_{z_1}^2 + 2\sigma_{z_2}^2$

$k = 2$:

$\mathbb{E}[z_1^2 z_2^4] = \underbrace{\mathbb{E}[z_1 z_2]}_{c} + \underbrace{\mathbb{E}[z_1 z_2]}_{c} + \underbrace{\mathbb{E}[z_2^2]}_{\sigma_{z_2}^2} = 2c + \sigma_{z_2}^2 = 10$

$\cancel{2c = 1 \quad c = \frac{10 - \sigma_{z_2}^2}{2} = \frac{10 - 1}{2} = 4{,}5 = cov(z_1, z_2)}$

$2c + \sigma_{z_1}^2 + 3\sigma_{z_2}^2 = 10$

$c = \frac{10 - \sigma_{z_1}^2 - 3\sigma_{z_2}^2}{2} = \frac{10 - 5}{2} = 2{,}5 = cov(z_1, z_2)$

Матрица ковариации $C = \begin{pmatrix} \sigma_{z_1}^2 & cov(z_1, z_2) \\ cov(z_1, z_2) & \sigma_{z_2}^2 \end{pmatrix} = \begin{pmatrix} 2 & 2{,}5 \\ 2{,}5 & 1 \end{pmatrix}$

№4.

$(X_1, X_2, X_3 \dots X_n)$ - выборка из распределения Парето

$$p(x|\alpha) = \frac{\alpha}{x^{\alpha+1}} \quad x \geq 1$$

$$L(\alpha) = \prod_i^n \frac{\alpha}{x_i^{\alpha+1}}$$

$$\ell(\alpha) = \ln L(\alpha) = \ln \frac{\alpha^n}{\prod_i^n x_i^{\alpha+1}} = n \ln \alpha - (\alpha+1) \sum_i^n \ln x_i$$

$$\frac{\partial \ell}{\partial \alpha} = \frac{n}{\alpha} - \sum_i^n \ln x_i = 0 \Rightarrow \frac{n}{\alpha_{max}} = \sum_i^n \ln x_i \Rightarrow \alpha_{max} = \frac{n}{\sum_i^n \ln x_i}$$

$$\frac{\partial^2 \ell}{\partial \alpha^2} = -\frac{n}{\alpha^2} < 0 \Rightarrow \frac{\partial \ell}{\partial \alpha} = 0 \Rightarrow \alpha_{max} - max$$

№3.

Z_1, Z_2, Z_3 независимы $\quad p(z) = Az \quad 0 \leq z \leq 2$

$$\cancel{1 = \int_0^2 dz_1 \int_0^2 dz_2 \int_0^2 dz_3 (Az) \Rightarrow \frac{1}{A} = \int_0^2 dz_1 \int_0^2 dz_2\, 2 \Rightarrow}$$

$$\cancel{\Rightarrow \frac{1}{A} = \int 2 \cdot 2 \cdot 2 \Rightarrow A = \frac{1}{8}}$$

$$1 = \int_0^2 Az\, dz \Rightarrow \frac{1}{A} = 2 \Rightarrow A = \frac{1}{2}$$

№1. Андронов Игорь БФЗ 241 Вариант - 50

(X, Y) $\hat{C} = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix} \Rightarrow$ $cov(X,Y) = 1$, $\sigma_x^2 = 4$ $\sigma_y^2 = 2$

$Z = X\cos\varphi + Y\sin\varphi$ $V = Y\cos\varphi - X\sin\varphi$

$\sigma^2(Z) = \sigma^2(X\cos\varphi + Y\sin\varphi) = \cos^2\varphi\,\sigma^2(X) + \sin^2\varphi\,\sigma^2(Y) + 2\cos\varphi\sin\varphi\, cov(X,Y) =$

$= 4\cos^2\varphi + 2\sin^2\varphi + 2\cos\varphi\sin\varphi$

$\sigma^2(V) = \sigma^2(Y\cos\varphi - X\sin\varphi) = \cos^2\sigma^2(Y) - 2\cos\varphi\sin\varphi + \sin^2\varphi\,\sigma^2(X) = 2\cos^2\varphi - 2\cos\varphi\sin\varphi + 4\sin^2\varphi =$

$cov(X,Y) = 1$

$= 2(\cos^2\varphi + \sin^2\varphi) - 2\sin 2\varphi + 2\sin^2\varphi = 2\sin^2\varphi - \sin 2\varphi + 2$

$\sigma^2(V) \geq 0 \Rightarrow 2\sin^2\varphi - \sin 2\varphi + 2 \geq 0$ верно при $\forall\varphi$

$\sigma^2(Z) = 4\cos^2\varphi + 2\sin^2\varphi + 2\cos\varphi\sin\varphi = 2 + 2\cos^2\varphi + 2\cos\varphi\sin\varphi =$

$= 2\cos^2\varphi + \sin 2\varphi + 2 \geq 0$ верно при $\forall\varphi$, т.к.

$\begin{cases} 2\cos^2\varphi \geq 0 \\ 2\cos^2\varphi \leq 2 \end{cases} + \begin{cases} \sin 2\varphi \leq 1 \\ \sin 2\varphi \geq -1 \end{cases} + 2 > 0$

Аналогично для $\sigma^2(V)$