Курмаметов Ганель

В-49.

№2.

$\langle X^6 \rangle = \frac{5}{9}$ $\quad$ X и Y распред по Гауссу

$\langle Y^4 \rangle = 3$ $\quad$ $\langle x \rangle = 0$

$\langle X^2 Y^4 \rangle = 3$ $\quad$ $\langle y \rangle = 0.$

исп. ф-лу вика

$$\langle x\,x\,x\,x\,x\,x \rangle = 15 \langle x^2 \rangle^3 = \frac{5}{9}$$

$(6-1)!! = 5!! = 15$ способ x_i разместить по парам

$$\Rightarrow \langle x^2 \rangle^3 = \frac{1}{27}$$

$$\Rightarrow \langle x^2 \rangle = \frac{1}{3} \Rightarrow \boxed{\sigma_x^2 = \frac{1}{3}}$$

$$\langle y^4 \rangle = \langle y\,y\,y\,y \rangle = 3 \langle y^2 \rangle^2 = 3$$

$$\Rightarrow \langle y^2 \rangle = 1. \quad \Rightarrow \boxed{\sigma_y^2 = 1}$$

$\sigma_y = -1$ — пост. т.к. σ_y^2

$$\langle x^2 y^4 \rangle = \langle x\,x\,y\,y\,y\,y \rangle = 3 \langle x^2 \rangle \langle y^2 \rangle^2 + 12 \langle xy \rangle^2 \langle y^2 \rangle$$

1. $\langle x^2 \rangle \langle y^2 \rangle^2$

1) $\langle x^2 \rangle$ — 1 способ разместить 2 x по парам

2) $\langle y^2 \rangle^2$ — $(4-1)!! = 3$ сп разм 4 y по парам.

$\Rightarrow 1 \cdot 3 = 3$ сп.

$3 \langle x^2 \rangle \langle y^2 \rangle^2$

2. $\langle xy \rangle^2 \langle y^2 \rangle$

1) $\langle xy \rangle^2$

для x: $C_4^2 = \frac{4!}{2!\,2!} = 6$ $\Rightarrow 6 \cdot 2 = 12$ сп.

для y: $2! = 2$

$\langle y^2 \rangle$ — 1 сп.

$\Rightarrow 12 \langle xy \rangle^2 \langle y^2 \rangle$

$$\Rightarrow 3 \cdot \frac{1}{3} \cdot 1 + 12 \langle xy \rangle^2 \cdot 1 = 3$$

$$\Rightarrow \langle xy \rangle^2 = \frac{1}{6} \Rightarrow \langle xy \rangle \frac{1}{\sqrt{6}}.$$

$$\Rightarrow \operatorname{cov}(X,Y) = \frac{1}{\sqrt{6}}$$

$$\Rightarrow r(X,Y) = \frac{1}{\sqrt{6}}\sqrt{3} = \frac{1}{\sqrt{2}} \qquad \boxed{r(X,Y) = \frac{1}{\sqrt{2}}}$$

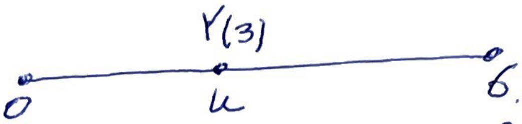
N3.

Y_1, Y_2, Y_3, Y_4

$$f(y_i) = \frac{1}{6}$$

$$F(y) = \frac{y}{6}$$

$$Y_{(1)} \leq Y_{(2)} < Y_{(3)} < Y_{(4)}$$



$$f_{Y(3)}(u) = n\, C_3^2 [F(u)]^2 [1-F(u)]^1 f(u)$$

$$\Rightarrow f_{Y(3)}(u) = \frac{4 \cdot 3 \cdot u^2}{36}\left(1-\frac{u}{6}\right)\cdot\frac{1}{6}$$

$$= \frac{u^2(6-u)}{18}$$

$$\cancel{P(1<[Y_{(3)}-3]^2<9) = \int_1^9 \left(\frac{u^2(6-u)}{18}-3\right)^2 du}$$

$$\cancel{= \int_1^9 \left(\frac{u^4(6-u)^2}{18^2} - \frac{u^2(6-u)}{3} + 9\right) du}$$

$$\cancel{Z = (Y_{(3)}-3)^2 \Rightarrow Y_{(3)} = Z+3}$$

$$\cancel{\Rightarrow f_Z(z) = \frac{(z+3)^2(3-z)}{18}}$$

$$\cancel{P(1<Z^2<9)}$$

Неуравновешенная задача В – 49

№4.

$$P(x|\alpha) = \frac{\alpha^K x^{K-1} e^{-\alpha x}}{\Gamma(K)}$$

$$L(\alpha | x_1, x_2, \dots, x_n) = \prod_{1\le i\le n} P(x_i|\alpha) = \frac{\alpha^{nK}}{\Gamma^n(K)} e^{-\alpha \sum_i x_i} \prod_{1\le i\le n} x_i^{K-1}$$

$$\ell = \ln L$$

$$\Rightarrow \ell = n \ln \frac{\alpha^K}{\Gamma(K)} - \alpha \sum_i x_i + (K-1) \sum_i \ln x_i$$

$$\frac{d\ell}{d\alpha} = \frac{nK}{\alpha} - \sum_i x_i = 0.$$

$$\frac{d^2\ell}{d\alpha^2} = -\frac{nK}{\alpha^2} = -\left(\sum_i x_i\right)^2 < 0$$

$$\Rightarrow \boxed{\alpha = \frac{nK}{\sum_i x_i}}$$

№1.

для вектора (X, Y) $\quad \hat{C} = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$ $\quad$ Z – незав. случ. вел.

$\Rightarrow \sigma_x^2 = 3$

$\sigma_Y^2 = 2$ $\quad cov(X,Y) = -1$ $\quad \hat{C} = \begin{pmatrix} \sigma^2(x) & cov(x,y) \\ cov(x,y) & \sigma^2(y) \end{pmatrix}$

$E[XY] - E[X]E[Y] = -1$

$X' = X + Z$

$Y' = Y + Z$ $\quad cov(X', Y') = 0 \Rightarrow E[X'Y'] - E[X']E[Y'] = 0$

$\sigma^2_{X'}, \sigma^2_{Y'}, \sigma^2_Z = ?$

$$\Rightarrow E[(X+z)(y+z)] - E(X+z)E(y+z) = 0$$

$$E[xy] + E[z^2] + E[Xz] + E[yz] - E[x]E[Y] - E^2[z] - E[x]E[z] - E[y]E[z]$$

т.к. незав.

$$\Rightarrow -1 + E[z^2] + \cancel{E[x]E[z]} + \cancel{E[y]E[z]} - \cancel{E[y]E[z]} - \cancel{E[x]E[z]} - E^2[z] = 0$$

$$\Rightarrow \boxed{\sigma_Z^2 = E[z^2] - E^2[z] = 1.}$$

$$\sigma_x^2 = E[x^2] - E^2[x] = E[(x'-z)^2] - E[x'-z] \quad E[x'$$

$$= E[x'^2] - 2E[x'z] + E[z^2] - E^2[x'] - E^2[z^2] +$$

$$= \sigma_{x'}^2 - 2E[x]E[z] - E[z^2] + 2E[x]E[z] + 2E^2[x] \quad 2E[x'z] \quad E[z$$

$$\sigma_{x'}^2 = E[x'^2] - E^2[x'] = E[(x+z)^2] - E^2[x+z]$$

$$= E[x^2] + 2E[x]E[z] + E[z^2] - E^2[\cancel{x}] - 2E[x]E[z] + E^2[z]$$

$$= 3 + 1 = 4$$

$$\sigma_{y'}^2 = E[y'^2] - E^2[y'] = E[y^2] - E^2[y]$$

$$+ E[z^2] - E^2[z] = 2 + 1 = 3$$