Федорова
БФ-241

№4 $(x_1, x_2, \dots, x_n)$

$$p(x|\alpha) = \frac{\alpha}{x^{\alpha+1}}, \quad x \geq 1$$

Оценка α – ?

$$L(\Theta | x_1, x_2, \dots, x_n) = \prod_{1 \leq i \leq n} p(x_i | \Theta)$$

$\ln L(\Theta | \vec{x})$ по непрерывности $\ln()$

$$\ln L(\Theta | \vec{x}) = \ln F(\Theta | \vec{x}) = \ln\left(\prod p(x_i | \Theta)\right) \ominus$$

пусть $\alpha \overset{def}{\equiv} F()$
$\Theta \overset{def}{\equiv} \alpha$

$$\ominus \ln\left(\prod_{1 \leq i \leq n} \frac{\alpha}{x_i^{\alpha+1}}\right) = \sum_{1 \leq i \leq n} \ln\left(\frac{\alpha}{x_i^{\alpha+1}}\right) = \sum_{1 \leq i \leq n} \ln \alpha - \sum_{1 \leq i \leq n} \ln(x_i^{\alpha+1}) =$$

$$= n \ln \alpha - \sum_{1 \leq i \leq n} (\alpha+1) \ln x_i = n \ln \alpha - (\alpha+1) \sum_{1 \leq i \leq n} \ln x_i$$

$$\frac{\partial F(\alpha | x_1, \dots, x_n)}{\partial \alpha} = \frac{n}{\alpha} - \sum_{1 \leq i \leq n} \ln x_i = 0$$

$$\frac{n}{\alpha} = \sum \ln x_i$$

$$\boxed{\alpha^* = \frac{n}{\sum_{1 \leq i \leq n} \ln x_i}}$$

Проверка на максимум:

$$\frac{\partial^2 F(\alpha | x_1, \dots, x_n)}{\partial \alpha^2} = -\frac{n}{\alpha^2} < 0 \Rightarrow \alpha^* \text{ – макс.}$$

Ответ $\boxed{\frac{n}{\sum_{1 \leq i \leq n} \ln x_i}}$

№2 $\vec{Z} = (Z_1, Z_2)$ $\mu = 0$ $\Rightarrow$ $\mathbb{E}[Z_1] = 0$
$\mathbb{E}[Z_2] = 0$

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

$$\langle Z_1^4 \rangle = 12$$
$$\langle Z_2^6 \rangle = 15$$
$$\langle Z_1^2 Z_2^4 \rangle = 10$$

Ф-ла Вика:
N - четн: $\langle X_1 \dots X_{2N} \rangle = \sum \prod C_{ij}$
все попарные спаривания

Следств-е форм Вика:

$$\langle Z_1^n \rangle = (2n-1)!! \langle Z_1^2 \rangle^n = (2n-1)!! \sigma^{2n}$$

$$\langle Z_1^4 \rangle = 3!! \langle Z_1^2 \rangle^4 = 3 \langle Z_1^2 \rangle^4 = 12$$

$$\langle Z_1^2 \rangle^4 = 4$$

$$\langle Z_1^2 \rangle = \sqrt[4]{4}$$

$$\langle Z_2^6 \rangle = 5!! \langle Z_2^2 \rangle^6 = 5 \cdot 3 \cdot 1 \langle Z_2^2 \rangle^6 = 15 \langle Z_2^2 \rangle^6 \overset{!}{=} 15$$

$$\langle Z_2^2 \rangle = 1$$

Либо 0 либо 2 пара $\langle Z_1 Z_2 \rangle$:

1сл: $\langle Z_1^2 Z_2^4 \rangle = \langle Z_1 Z_1 \rangle + 2 \langle Z_2 Z_2 \rangle = \sqrt[4]{4} + 2$

2сл: $\langle Z_1^2 Z_2^4 \rangle = 2 \langle Z_1 Z_2 \rangle + \langle Z_2 Z_2 \rangle = 2\langle Z_1 Z_2 \rangle + 1$

$$C_{ij} = \mathbb{E}[Z_i Z_j] - \mathbb{E} Z_i \, \mathbb{E} Z_j = \mathbb{E}[Z_i Z_j]$$

$$C_{ii} = \mu_2 \qquad \hat{C} = \begin{pmatrix} \sqrt[4]{4} & \cdot \\ \cdot & 1 \end{pmatrix}$$

$$C_{12} = \mathbb{E}[Z_1 Z_2]$$

$$r(Z_1, Z_2) = \frac{\mathbb{E}[Z_1 Z_2]}{\sigma_{Z_1} \sigma_{Z_2}} = \frac{\mathbb{E}[Z_1 Z_2]}{\sqrt[4]{4}}$$

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1сл + 2сл =>

1сл: $z_1 z_1 \; z_2 z_2 z_2 z_2$

$\frac{2\cdot 1}{2}\cdot 1 = 1$ способ

2сл: $z_1 z_1 \; z_2 z_2 z_2 z_2$: $2\langle z_1 z_2\rangle + \langle z_2^2\rangle$

$C_2^1 \, C_4^2 = \frac{4!}{2!\,2!} = 3$ способ

Распишем эти 3 способа

$$\sqrt[4]{4} + 2 + 2\langle z_1 z_2\rangle + 1 = 10$$

$$\langle z_1 z_2\rangle = \frac{7-\sqrt[4]{4}}{2}$$

$$\boxed{\hat{C} = \begin{pmatrix} \sqrt[4]{4} & \frac{7-\sqrt[4]{4}}{2} \\ \frac{7-\sqrt[4]{4}}{2} & 1 \end{pmatrix}}$$

N1 (X, Y)

$$\hat{C} = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} \langle X^2 \rangle & \\ & \langle Y^2 \rangle \end{pmatrix}$$

$$\begin{cases} Z = X\cos\varphi + Y\sin\varphi \\ V = Y\cos\varphi - X\sin\varphi \end{cases}$$ – некор $\Longleftrightarrow$

φ – ?

$\mathbb{D}Z$ – ? $\mathbb{D}V$ – ?

$\operatorname{cov}(Z, V) = 0$;

$$\Longleftrightarrow \quad r(Z, V) = \frac{\operatorname{cov}(Z, V)}{\sigma_Z \sigma_V} = 0$$

$$\mathbb{E}[ZV] = \mathbb{E}[Z]\,\mathbb{E}[V].$$

$$\langle Z \rangle = \langle X\cos\varphi \rangle + \langle Y \sin\varphi \rangle =$$

т.к. φ = const (параметр) при разл. X, Y

$$= \cos\varphi \langle X \rangle + \sin\varphi \langle Y \rangle$$

$$\mathbb{D}X = \langle X^2 \rangle = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

$$\langle V \rangle = \cos\varphi \langle Y \rangle - \sin\varphi \langle X \rangle$$

(1) $$(\langle Z \rangle)^2 = (\langle X \rangle)^2 + (\langle Y \rangle)^2 + 2\sin\varphi\cos\varphi \langle X \rangle \langle Y \rangle$$

$$(\langle V \rangle)^2 = (\langle Y \rangle)^2 + (\langle X \rangle)^2 - 2\cos\varphi\sin\varphi \langle X \rangle \langle Y \rangle$$

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$$(\langle Z\rangle)^2+(\langle V\rangle)^2 = 2\left((\langle X\rangle)^2+(\langle Y\rangle)^2\right) =$$

$$= 2(4+2) = 12$$

$$\mathbb{D}Z = \mathbb{E}[Z^2]-(\mathbb{E}[Z])^2 \cancel{=}$$

$$\cancel{=} \quad (\langle Z\rangle)^2 = 12-(\langle V\rangle)^2$$

Найдем $\langle X\rangle$; $\langle Y\rangle$

$$\hat{C} = \begin{pmatrix} \underset{\parallel}{\langle X^2\rangle}^{4} & \underbrace{\mathbb{E}[X_1 X_2]-\mathbb{E}X_1\mathbb{E}X_2}_{} \overset{1}{\parallel} \\ \mathbb{E}[X_1X_2]-\mathbb{E}[X_1]\mathbb{E}[X_2] & \underset{\underset{2}{\parallel}}{\langle Y^2\rangle} \end{pmatrix}$$

$$(\mathbb{E}Z+\mathbb{E}V)^2 = (\mathbb{E}Z)^2+(\mathbb{E}V)^2+2\underbrace{\mathbb{E}Z\,\mathbb{E}V}_{\mathbb{E}[ZV]}$$

$$2\left((\langle X\rangle)^2+(\langle Y\rangle)^2\right) \cancel{=} 2\mathbb{E}\left[(X\cos\varphi+Y\sin\varphi)(Y\cos\varphi-X\sin\varphi)\right] =$$

$\cancel{=}$

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N 3

$z_1 \perp z_2 \perp z_3$ iid

$$\begin{cases} p(z) = Az, & 0 \le z \le 2 \\ p(z) = 0, & \text{иначе} \end{cases}$$

A – ?

$$\int_{-\infty}^{+\infty} Az\,dz = 1$$

$$A\int_0^2 z\,dz = 1$$

$$A\left.\frac{z^2}{2}\right|_0^2 = 2A = 1 \Rightarrow A = \frac{1}{2}$$

$\mathbb{E}[\max z]$

$$\mathbb{P}(\underset{z}{\max\{z_1, z_2, z_3\}}) = \mathbb{P}((z_1 \le z, z_2 \le z, z_3 \le z)) =$$

$$= \cancel{\mathbb{F}(\max\{\ldots\} \le z)} = \cancel{(F())^3} \; (F(z_i \le z))^3 = \mathbb{F}(\text{maximum})$$

~~Для макс:~~ $\cancel{\frac{dF()}{dz} = p(z)}$

обычная (не для max)

$$\frac{dF}{dz} = p(z)$$

Для макс: $\frac{dF^3()}{dz} = p_{\text{макс}}(z)$

$$p_{\text{макс}}(z) = 3F^2()F'() \quad ; \quad \cancel{\text{обычная } F = \int_0^2 p(z)dz = \frac{1}{2}}$$

$$\cancel{p_{\text{макс}}(z) = 3 \cdot \frac{1}{4} \cdot 0}$$

обычная $F = \int_{-\infty}^{z} p(z)dz = \int_{-\infty}^{z} Az\,dz = \frac{Az^2}{2}$

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