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Вариант 45

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$\langle x^6 \rangle = \frac{5}{9}$ $\quad \langle y^4 \rangle = \langle x^2 y^4 \rangle = 3 \quad \langle x \rangle = \langle y \rangle = 0 \quad r - ?$

$r = \frac{\mathrm{cov}(x,y)}{\sigma_x \sigma_y} \qquad \mathbb{E}[x^{2n}] = (2n-1)!!\,\sigma^{2n} \qquad \mathbb{E}[x\overset{4}{y}] = \mathrm{cov}(x,y)$

- σ_x: $n=3 \Rightarrow 15\sigma_x^6 = \frac{5}{9} \Rightarrow \sigma_x^6 = \frac{1}{27} \Rightarrow \sigma_x^2 = \frac{1}{3} \Rightarrow \sigma_x = \frac{1}{\sqrt{3}}$

  σ_y: $n=2 \Rightarrow 3\sigma_y^4 = 3 \Rightarrow \sigma_y^4 = 1 \Rightarrow \sigma_y = 1$

- $\langle xy \rangle$ из $\langle x^2 y^4 \rangle$ с помощью Вика:

$a = \langle xx \rangle = \frac{1}{3}$
$b = \langle yy \rangle = 1$
$c = \langle xy \rangle = \mathrm{cov}(x,y)$

xy	xy
xy	yy
yy	yy
1	2

1 $c^2 b \cdot 4 \cdot 3 \cdot 1 = 12c^2 b$

$x_1 x_2$
$y_1 y_2 y_3 y_4$

2 $ab^2 \cdot 1 \cdot 3 = 3ab^2$

$y_1 y_2$ и $y_3 y_4$ 1
$y_1 y_3$ $y_2 y_4$ 2
14 23 3

$\langle x^2 y^4 \rangle = 12c^2 b + 3ab^2 = 12c^2 + 1 = 3$

$c^2 = \frac{2}{12} = \frac{1}{6}$

$\underbrace{c = \frac{1}{\sqrt{6}} = \langle xy \rangle}$

- $r = \frac{\frac{1}{\sqrt{6}}}{\frac{1}{\sqrt{3}} \cdot 1} = \sqrt{\frac{1}{2}} = \boxed{\frac{\sqrt{2}}{2}}$

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$p(x|\alpha) = \alpha^k x^{k-1} e^{-\alpha x}/\Gamma(k) \quad x \geq 0 \qquad \hat{\alpha}_{ML} - ? \qquad k$ известно

$L(\alpha|\{x\}) = \prod_{1 \leq i \leq n} p(x|\alpha) \qquad \hat{\alpha}_{ML} = \arg\max_{\alpha} L(\alpha|\{x\}) \qquad \mathcal{L}(\alpha|x) \equiv \ln L(\alpha|\{x\})$

$L(\alpha|\{x\}) = \frac{\alpha^k}{\Gamma(k)} \left(\prod_i x_i^{(k-1)}\right) \cdot e^{-\alpha \sum x_i} \Rightarrow \mathcal{L}(\alpha|\{x\}) =$

$= \ln L(\alpha|\{x\}) = k \ln \frac{\alpha}{\Gamma(k)^{1/k}} + \sum \ln x_i^{(k-1)} - \alpha \sum x_i$

$\mathcal{L}'_\alpha = \frac{k}{\alpha} - \sum x_i = 0 \Rightarrow \hat{\alpha}_{ML} = \frac{k}{\sum x_i} \qquad \mathcal{L}''_{\alpha\alpha} = -\frac{k}{\alpha^2} < 0$ при $k > 0 \Rightarrow \max$

$\boxed{\hat{\alpha}_{ML} = \frac{k}{\sum x_i}}$

N1 →

M]

$$\hat{C} = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} D(x) & cov(x,y) \\ cov(x,y) & Dy \end{pmatrix}$$

$$r' = 0 = \frac{cov(x',y')}{\sigma_{x'}\sigma_{y'}} \Rightarrow$$

$$\Rightarrow cov(x',y') = 0$$

$D(x) = 3 = cov(x,x) = \mathbb{E}[x^2] - (\mathbb{E}[x])^2$

$D(y) = 2 = cov(y,y) = \mathbb{E}[y^2] - (\mathbb{E}[y])^2$

$cov(x,y) = -1 = \mathbb{E}[xy] - \mathbb{E}[x]\,\mathbb{E}(y)$

$cov(x+y,z) = cov(x,z) + cov(y,z)$ $\quad \mathbb{E}[x+z] = \mathbb{E}[x] + \mathbb{E}[z]$

$cov(x'y') = 0 \Rightarrow \mathbb{E}[x'y'] - \mathbb{E}[x']\mathbb{E}[y'] = 0$

$cov(x,y) = -1$ $\quad \mathbb{E}[xy + xz + zy + z^2] - \mathbb{E}[x]\mathbb{E}[y] - \mathbb{E}[x]\mathbb{E}[z] - \mathbb{E}[z]\mathbb{E}[y] - \mathbb{E}[z]^2 = 0$

$\mathbb{E}[xy] + \mathbb{E}[xz] + \mathbb{E}[zy] + \mathbb{E}[z^2] - \mathbb{E}[x]\mathbb{E}[y] - \mathbb{E}[x]\mathbb{E}[z] - \mathbb{E}[z]\mathbb{E}[y] - \mathbb{E}[z]^2$

$D(z)$

$-1 + cov(x,z) + cov(z,y) + D(z) = 0$

$\underset{\text{тк незав}}{cov(x,z) + cov(z,y)} = \mathbb{E}[x]\mathbb{E}[z] - \mathbb{E}[x]\mathbb{E}[z] + \mathbb{E}[x]\mathbb{E}[z] - \mathbb{E}[x]\mathbb{E}[z] = 0$

$$\boxed{D(z) = 1}$$

$D(x+z) = D(x) + D(z) + 2D(x)D(z) = 3 + 1 + 2 \cdot 3 \cdot 1 = 10$

$D(y+z) = D(y) + D(z) + 2D(y)D(z) = 2 + 1 + 2 \cdot 2 \cdot 1 = 7$

$$\boxed{\begin{aligned} D(z) &= 1 \\ D(x') &= 10 \\ D(y') &= 7 \end{aligned}}$$