Васильев М.Н. БФ3243 Вариант №43

№3

$\overline{Y_1, Y_n}$ iid $\qquad F(y) = y/6$

$p_{Y_i}(y) = 1/6 \quad y \in [0, 6]$

$p_{(3)}(y) = 4\binom{3}{2}\frac{y^2}{36}\left[1 - y/6\right] 1/6 = \frac{y^2}{18}\left[1 - y/6\right]$

Определим обл опр y, через усл. нормировки

$\int_0^x \frac{y^2}{18}\left[1 - y/6\right] dy = \frac{x^3}{18 \cdot 3} - \frac{x^4}{6 \cdot 4 \cdot 18} = 1$

$-\frac{x^4}{24 \cdot 18} + \frac{x^3}{54} - 1 = 0$

$\frac{x^4}{18} - \frac{4x^3}{9} + 24 = 0$ ~~Рац. корней нет~~

$x^4 - 8x^3 + 432 = 0$

$P(1 < [Y_{(3)} - 3]^2 < 9) = \int_1^9 (p(y-3))^2 dy$ ...

№2

X, Y – распр. по Гауссу с $\mu = 0$

~~$\langle X^6 \rangle = \binom{6}{2}\langle X^2 \rangle = 15 \langle X^2 \rangle = 5/9$~~

$\langle Y^4 \rangle = \cancel{\binom{4}{2}\langle Y^2 \rangle} = 3\langle Y^2 \rangle^2 = 3\,\mathrm{var}^2(Y) = 3 \Rightarrow \cancel{\mathrm{var}(X) = 1}$

$\langle X^6 \rangle = 5\langle X^2 \rangle^3 + 5\langle X^2 \rangle\langle X^4 \rangle = 5\langle X^2 \rangle^3 + 15\langle X^2 \rangle^3 =$

$= 20\underset{\mathrm{var}(X)}{\langle X^2 \rangle^3} = 5/9$

$\langle X^2 Y^4 \rangle =$

$\overline{X_1, X_n}$ iid

№4

$$p(x|\alpha) = \frac{\alpha^k x^{k-1} e^{-\alpha x}}{\Gamma(k)} \quad x \geq 0$$

$$\mathcal{L}(\alpha|\vec{x}) = \prod_{1 \leq i \leq n} \frac{\alpha^k x_i^{k-1} e^{-\alpha x}}{\Gamma(k)}$$

$$L(\alpha|\vec{x}) = \sum_{i=1}^{n} k \ln\alpha + (k-1)\ln x_i - \alpha x - \ln\Gamma(k)$$

$$\cancel{L_\alpha'(\alpha|\vec{x})} = L(\alpha|\vec{x}) = k\ln\alpha + (k-1)\ln x - \alpha x - \ln\Gamma(k)$$

$$\cancel{L_\alpha'(\alpha|\vec{x}) = -\alpha + \frac{(k-1)n}{x^n} = 0} \qquad L_\alpha' = k/\alpha - x = 0$$

$$\cancel{\alpha = \frac{(k-1)n}{x^n}} \qquad \boxed{\alpha = k/x}$$

№1

$$(X, Y) \qquad \hat{C} = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$$

$$X' = X + Z \qquad cov(X'Y') = 0$$
$$Y' = Y + Z$$

Z – незав.

$$cov(XY) = E[XY] - E[X]E[Y] = -1$$

$$cov(X'Y') = E[(X+Z)(Y+Z)] - E[X+Z]E[Y+Z] =$$
$$= E[XY] + E[Z^2] + \cancel{E[XZ]} + \cancel{E[YZ]} - (E[X]E[Y] + E^2[Z] + \cancel{E[X]E[Z]} + E[Y]E[Z]) =$$

$$= \underbrace{(E[XY] - E[X]E[Y])}_{-1} + \underbrace{E[Z^2] - E^2[Z]}_{D(Z)} = 0$$

$$D(Z) = 1$$

$$D(X') = D(X+Z) = D(X) + D(Z) = 3 + 1 = 4$$

$$D(Y') = D(Y) + D(Z) = 2 + 1 = 3$$