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БФ3241

В-41

№2 $\langle x^6\rangle = \frac{5}{9}$; $\langle y^4\rangle = 3$; $\langle x^2y^4\rangle = 3$; $\tau = \frac{cov(x,y)}{\sigma_x\sigma_y} = ?$

1) $\langle x^{2k}\rangle = (2k-1)!!\cdot(\sigma_x)^{2k}$

$cov(x,y) = \langle xy\rangle - \underbrace{\langle x\rangle\langle y\rangle}_{0}$
$\overset{\shortparallel}{c}$

$\Rightarrow \langle x^6\rangle = \cancel{5}\cdot 3\cdot\sigma_x^6 = \frac{5}{9} \Rightarrow \sigma_x^6 = \frac{1}{27}$

$\sigma_x^2 = \langle x^2\rangle - \underbrace{\langle x\rangle^2}_{0} = \langle x^2\rangle$

$\langle y^4\rangle = 3\cdot\sigma_y^4 = 3 \Rightarrow \sigma_y = 1$

Th. Вика

2) $\langle x^2y^4\rangle$: I) $\langle \underbrace{xy\cdot xy}_{2\text{сп.}}\cdot\underbrace{y\cdot y}_{4\cdot 3\text{ способов}}\rangle =$

$= \langle x^2y^2\rangle\langle xy^3\rangle + \langle xy^2\rangle\langle xy^2\rangle + \langle xy^2\rangle\langle xy^2\rangle =$

$= \langle x^2\rangle\cdot\langle y^2\rangle + \langle xy\rangle\langle y^2\rangle + \langle xy\rangle\langle xy\rangle + 0 + 0 = \sigma_x^2\sigma_y^2 + c\sigma_y^2 +$
$+ c^2$

$\Rightarrow 24(\sigma_x^2\sigma_y^2 + c\sigma_y^2 + c^2) = 3$

$24(\underbrace{\sqrt[3]{\frac{1}{27}}}_{\frac{1}{3}}\cdot 1 + c + c^2) = 3 \Rightarrow c^2 + c + \frac{1}{3} = \frac{3}{24}$

$c^2 + c + \frac{8-3}{24} = 0$

$c^2 + c + \frac{5}{24} = 0$

$D = 1 - \frac{4\cdot 5}{24} = 1 - \frac{20}{24} = \frac{24-20}{24} = \frac{4}{24} = \frac{1}{6}$

$\Rightarrow c = \frac{-1 \pm \frac{1}{\sqrt{6}}}{2} = \left[\begin{array}{l}\frac{-1-\frac{1}{\sqrt{6}}}{2}\\ \frac{-1+\frac{1}{\sqrt{6}}}{2}\end{array}\right.$

$\Rightarrow \tau = \left[\begin{array}{l}\frac{-1-\frac{1}{\sqrt{6}}}{2\cdot\frac{1}{\sqrt[6]{27}}}\\ \frac{-1+\frac{1}{\sqrt{6}}}{2\cdot\frac{1}{\sqrt[6]{27}}}\end{array}\right.$

№3 $[0,6]\sim Uniform$, $n=4$

1) $p_{(3)}(y) = 4\cdot C_3^2\cdot F^2(y)\cdot(1-F(y))\,p(y)$

$p(y) = \frac{1}{6} \Rightarrow F(y) = \frac{y}{6}$

$p_{(3)}(y) = \cancel{4}^{2}\cdot\frac{\cancel{6}}{\cancel{24}}\cdot\frac{y^2}{\cancel{36}_{18}}\cdot\left(1-\frac{y}{6}\right)\frac{1}{\cancel{6}} = \frac{y^2}{18}\left(1-\frac{y}{6}\right)$

~~2) $\mathbb{P}(1 < (Y_{(3)}-3)^2 < 9) = \int_4^6 (y-3)^2\,p_{(3)}(y)\,dy =$~~
~~$1 < Y_{(3)}-3 < 3 \Rightarrow Y_{(3)}\in(4,6)$~~

$$\cancel{\int_4^6 (y-3)^2 \cdot \frac{y^2}{18}\left(1-\frac{y}{6}\right)dy = \frac{1}{18}\int_4^6 (y^2-6y+9)y^2\left(1-\frac{y}{6}\right)dy =}$$

$$\cancel{= \frac{1}{18}\int_4^6} \qquad 1 < (y_{(3)}-3)^2 < 9 \Leftrightarrow y_{(3)} \in (0,2) \cup (4,6)$$

$$\mathbb{P}\left(1 < (y_{(3)}-3)^2 < 9\right) = Z\,\mathbb{P}\left(y_{(3)} \in (0,2) \cup (4,6)\right) =$$

$$= \int_0^2 \rho_{(3)}(y)\,dy + \int_4^6 \rho_{(3)}(y)\,dy = \frac{1}{18}\left[\int_0^2 y^2\left(1-\frac{y}{6}\right)dy + \right.$$

$$\left. + \int_4^6 y^2\left(1-\frac{y}{6}\right)dy\right] = \frac{1}{18}\left[\int_0^2 \left(y^2 - \frac{y^3}{6}\right)dy + \int_4^6 \left(y^2 - \frac{y^3}{6}\right)dy\right] =$$

$$= \frac{1}{18}\left[\frac{8}{3} - \frac{1}{6}\cdot \cancel{\frac{16}{4}}\ \frac{8}{21}\ + \frac{6^3}{3} - \frac{4^3}{3} - \frac{1}{6}\cdot\left(\frac{6^4}{4} - \frac{4^4}{4}\right)\right] =$$

$$= \frac{1}{18}\left[\frac{8}{3} - \frac{2}{3} + \frac{6\cdot 6\cdot 6^2}{3} - \frac{64}{3} - \frac{1}{6}\cdot\frac{1296-256}{4}\right] =$$

$$= \frac{1}{18}\left[2 + 72 - \frac{64}{3} - \frac{1040}{24}\right] = \frac{1}{18}\left[74 - \frac{64}{3} - \cancel{\ }\,35\right]$$

$$Z = \int_0^6 \rho_{(3)}(y)\,dy = \frac{1}{18}\int_0^6 y^2\left(1-\frac{y}{6}\right)dy = \frac{1}{18}\left[\frac{6^3}{3} - \frac{1}{6}\cdot\frac{6^{\cancel{4}\,3}}{4}\right] =$$

$$= \frac{1}{18}\left[\cancel{\ } 72 - \frac{216}{4}\right]$$

$$\Rightarrow \mathbb{P} = \frac{74 - \frac{64}{3} - 35}{72 - \frac{216}{4}} = \frac{39 - \frac{64}{3}}{72-54} = \frac{39 - \frac{64}{3}}{18}$$

№2 $\quad p(x|\alpha) = \dfrac{\alpha^k x^{k-1} e^{-\alpha x}}{\Gamma(k)}$, $x > 0$; k; α - ?

$$L = \prod_{i=1}^n \left(\frac{\alpha^k x_i^{k-1} e^{-\alpha x_i}}{\Gamma(k)}\right) = \left(\frac{\alpha^k}{\Gamma(k)}\right)^n \cdot \left(\prod_{i=1}^n (x_i)\right)^{k-1} \cdot e^{-\alpha \sum_{i=1}^n x_i}$$

$$\ln L = n \cdot \underbrace{\ln\left(\frac{\alpha^k}{\Gamma(k)}\right)}_{} + (k-1)\cdot \sum_{i=1}^n \ln(x_i) - \alpha \sum_{i=1}^n x_i$$

$$\cancel{\ln L}\ n\left(k\cdot \ln(\alpha) - \ln(\Gamma(k))\right)$$

$$(\ln L)'_{\alpha} = \frac{nk}{\alpha} - \sum_{i=1}^{n} x_i = 0 \Rightarrow \alpha = \frac{n \cdot k}{\sum_{i=1}^{n} x_i}$$

$$(\ln L)''_{\alpha\alpha} = -\frac{nk}{\alpha^2} < 0 \Rightarrow \max$$

№1

$$\hat{C} = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix} \Rightarrow \begin{array}{l} cov(x,x) = E(x^2) - E(x)^2 = 3 = \sigma_x^2 \\ \sigma_y^2 = 2 \\ cov(x,y) = E(xy) - E(x)E(y) = -1 \end{array}$$

$$(x, y) \mapsto (x+z, y+z)$$

Некорр. $\Rightarrow \rho = 0 \Rightarrow cov(x+z, y+z) = 0 \Rightarrow E((x+z)(y+z)) -$

$$- E(x+z)E(y+z) = E(xy + xz + yz + z^2) - (E(x) + E(z)) \cdot$$

$$\cdot (E(y) + E(z)) = 0$$

$$\Rightarrow E(xy) \cancel{+ E(xz)} + \cancel{E(yz)} + E(z^2) - E(x)E(y) \cancel{- E(x)E(z)} \cancel{- E(z)E(y)} -$$

$$- E(z)^2 = 0 \Rightarrow E(xy) + \underbrace{E(z^2) + E(z)^2}_{\sigma_z^2} - E(x)E(y) = 0$$

$$\sigma_z^2 = E(x)E(y) - E(xy) = 1$$

2) $\sigma_{x'}^2 = E((x+z)^2) - E(x+z)^2 = E(x^2) \cancel{+ 2E(x)E(z)} + E(z^2) -$

$$- E(x)^2 - \cancel{2E(x)E(z)} - E(z)^2 = \sigma_x^2 + \sigma_z^2 = 3 + 1 = 4$$

3) $\sigma_{y'}^2 = E(y^2) + E(z^2) - E(x)^2 - E(z)^2 = \sigma_y^2 + \sigma_z^2 = 2 + 1 = 3$